# 5 | ELECTRIC CHARGES AND FIELDS

**Figure 5.1** Electric charges exist all around us. They can cause objects to be repelled from each other or to be attracted to each other. (credit: modification of work by Sean McGrath)

## Chapter Outline



## Introduction

Back when we were studying Newton's laws, we identified several physical phenomena as forces. We did so based on the effect they had on a physical object: Specifically, they caused the object to accelerate. Later, when we studied impulse and momentum, we expanded this idea to identify a force as any physical phenomenon that changed the momentum of an object. In either case, the result is the same: We recognize a force by the effect that it has on an object.

In **Gravitation (http://cnx.org/content/m58344/latest/)** , we examined the force of gravity, which acts on all objects with mass. In this chapter, we begin the study of the electric force, which acts on all objects with a property called charge. The electric force is much stronger than gravity (in most systems where both appear), but it can be a force of attraction or a force of repulsion, which leads to very different effects on objects. The electric force helps keep atoms together, so it is of fundamental importance in matter. But it also governs most everyday interactions we deal with, from chemical interactions to biological processes.

# 5.1 | Electric Charge

## Learning Objectives

By the end of this section, you will be able to:

- Describe the concept of electric charge
- Explain qualitatively the force electric charge creates

You are certainly familiar with electronic devices that you activate with the click of a switch, from computers to cell phones to television. And you have certainly seen electricity in a flash of lightning during a heavy thunderstorm. But you have also most likely experienced electrical effects in other ways, maybe without realizing that an electric force was involved. Let's take a look at some of these activities and see what we can learn from them about electric charges and forces.

## Discoveries

You have probably experienced the phenomenon of **static electricity**: When you first take clothes out of a dryer, many (not all) of them tend to stick together; for some fabrics, they can be very difficult to separate. Another example occurs if you take a woolen sweater off quickly—you can feel (and hear) the static electricity pulling on your clothes, and perhaps even your hair. If you comb your hair on a dry day and then put the comb close to a thin stream of water coming out of a faucet, you will find that the water stream bends toward (is attracted to) the comb (**Figure 5.2**).

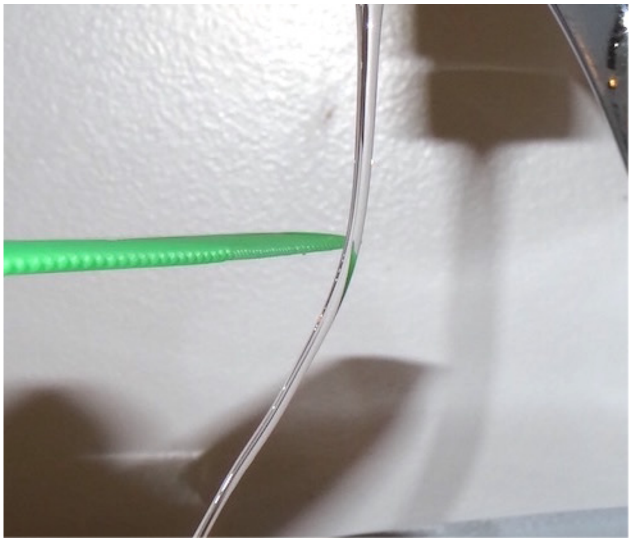


**Figure 5.2** An electrically charged comb attracts a stream of water from a distance. Note that the water is not touching the comb. (credit: Jane Whitney)

Suppose you bring the comb close to some small strips of paper; the strips of paper are attracted to the comb and even cling to it (**Figure 5.3**). In the kitchen, quickly pull a length of plastic cling wrap off the roll; it will tend to cling to most any nonmetallic material (such as plastic, glass, or food). If you rub a balloon on a wall for a few seconds, it will stick to the wall. Probably the most annoying effect of static electricity is getting shocked by a doorknob (or a friend) after shuffling your feet on some types of carpeting.

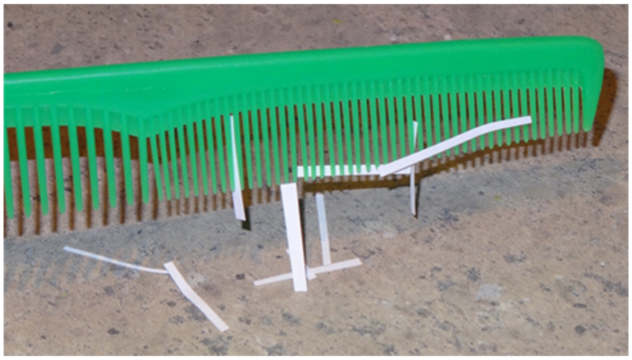


**Figure 5.3** After being used to comb hair, this comb attracts small strips of paper from a distance, without physical contact. Investigation of this behavior helped lead to the concept of the electric force. (credit: Jane Whitney)

Many of these phenomena have been known for centuries. The ancient Greek philosopher Thales of Miletus (624–546 BCE) recorded that when amber (a hard, translucent, fossilized resin from extinct trees) was vigorously rubbed with a piece of fur, a force was created that caused the fur and the amber to be attracted to each other (**Figure 5.4**). Additionally, he found that the rubbed amber would not only attract the fur, and the fur attract the amber, but they both could affect other (nonmetallic) objects, even if not in contact with those objects (**Figure 5.5**).

**Figure 5.4** Borneo amber is mined in Sabah, Malaysia, from shale-sandstone-mudstone veins. When a piece of amber is rubbed with a piece of fur, the amber gains more electrons, giving it a net negative charge. At the same time, the fur, having lost electrons, becomes positively charged. (credit: “Sebakoamber”/Wikimedia Commons)

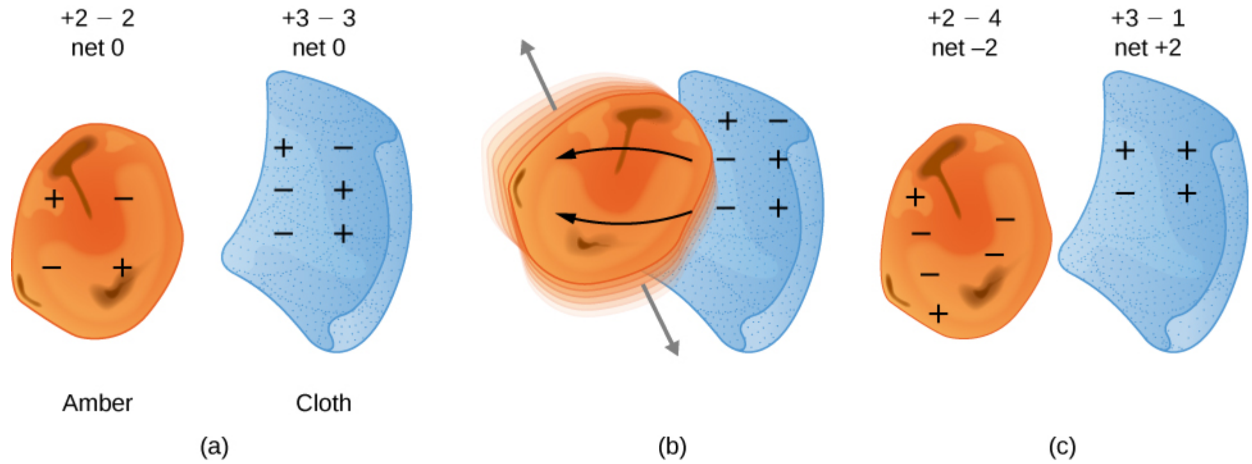



**Figure 5.5** When materials are rubbed together, charges can be separated, particularly if one material has a greater affinity for electrons than another. (a) Both the amber and cloth are originally neutral, with equal positive and negative charges. Only a tiny fraction of the charges are involved, and only a few of them are shown here. (b) When rubbed together, some negative charge is transferred to the amber, leaving the cloth with a net positive charge. (c) When separated, the amber and cloth now have net charges, but the absolute value of the net positive and negative charges will be equal.

The English physicist William Gilbert (1544–1603) also studied this attractive force, using various substances. He worked with amber, and, in addition, he experimented with rock crystal and various precious and semi-precious gemstones. He also experimented with several metals. He found that the metals never exhibited this force, whereas the minerals did. Moreover, although an electrified amber rod would attract a piece of fur, it would repel another electrified amber rod; similarly, two electrified pieces of fur would repel each other.

This suggested there were two types of an electric property; this property eventually came to be called **electric charge**. The difference between the two types of electric charge is in the directions of the electric forces that each type of charge causes: These forces are repulsive when the same type of charge exists on two interacting objects and attractive when the charges are of opposite types. The SI unit of electric charge is the **coulomb** (C), after the French physicist Charles-Augustin de Coulomb (1736–1806).

The most peculiar aspect of this new force is that it does not require physical contact between the two objects in order to cause an acceleration. This is an example of a so-called “long-range” force. (Or, as Albert Einstein later phrased it, “action at a distance.”) With the exception of gravity, all other forces we have discussed so far act only when the two interacting objects actually touch.

The American physicist and statesman Benjamin Franklin found that he could concentrate charge in a “ Leyden jar,” which was essentially a glass jar with two sheets of metal foil, one inside and one outside, with the glass between them (**Figure 5.6**). This created a large electric force between the two foil sheets.

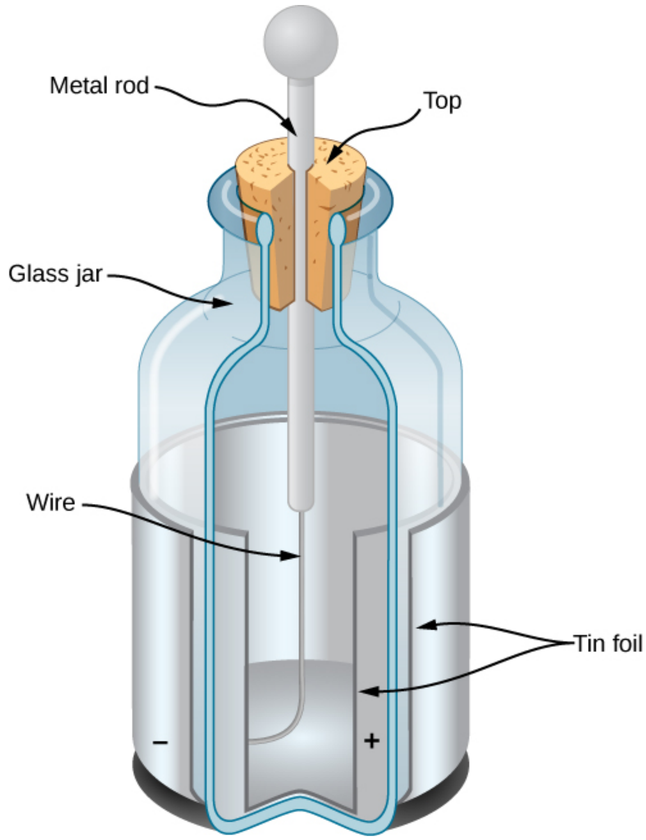



**Figure 5.6** A Leyden jar (an early version of what is now called a capacitor) allowed experimenters to store large amounts of electric charge. Benjamin Franklin used such a jar to demonstrate that lightning behaved exactly like the electricity he got from the equipment in his laboratory.

Franklin pointed out that the observed behavior could be explained by supposing that one of the two types of charge remained motionless, while the other type of charge flowed from one piece of foil to the other. He further suggested that an excess of what he called this "electrical fluid" be called "positive electricity" and the deficiency of it be called "negative electricity." His suggestion, with some minor modifications, is the model we use today. (With the experiments that he was able to do, this was a pure guess; he had no way of actually determining the sign of the moving charge. Unfortunately, he guessed wrong; we now know that the charges that flow are the ones Franklin labeled negative, and the positive charges remain largely motionless. Fortunately, as we'll see, it makes no practical or theoretical difference which choice we make, as long as we stay consistent with our choice.)

Let's list the specific observations that we have of this **electric force**:

- The force acts without physical contact between the two objects.
- The force can be either attractive or repulsive: If two interacting objects carry the same sign of charge, the force is repulsive; if the charges are of opposite sign, the force is attractive. These interactions are referred to as **electrostatic repulsion** and **electrostatic attraction**, respectively.
- Not all objects are affected by this force.
- The magnitude of the force decreases (rapidly) with increasing separation distance between the objects.

To be more precise, we find experimentally that the magnitude of the force decreases as the square of the distance between the two interacting objects increases. Thus, for example, when the distance between two interacting objects is doubled, the force between them decreases to one fourth what it was in the original system. We can also observe that the surroundings of the charged objects affect the magnitude of the force. However, we will explore this issue in a later chapter.

## Properties of Electric Charge

In addition to the existence of two types of charge, several other properties of charge have been discovered.

- **Charge is quantized.** This means that electric charge comes in discrete amounts, and there is a smallest possible amount of charge that an object can have. In the SI system, this smallest amount is $e \equiv 1.602 \times 10^{-19}\ \text{C}$. No free particle can have less charge than this, and, therefore, the charge on any object—the charge on all objects—must be an integer multiple of this amount. All macroscopic, charged objects have charge because electrons have either been added or taken away from them, resulting in a net charge.
- **The magnitude of the charge is independent of the type.** Phrased another way, the smallest possible positive charge (to four significant figures) is $+1.602 \times 10^{-19}\ \text{C}$, and the smallest possible negative charge is $-1.602 \times 10^{-19}\ \text{C}$; these values are exactly equal. This is simply how the laws of physics in our universe turned out.
- **Charge is conserved.** Charge can neither be created nor destroyed; it can only be transferred from place to place, from one object to another. Frequently, we speak of two charges "canceling"; this is verbal shorthand. It means that if two objects that have equal and opposite charges are physically close to each other, then the (oppositely directed) forces they apply on some other charged object cancel, for a net force of zero. It is important that you understand that the charges on the objects by no means disappear, however. The net charge of the universe is constant.
- **Charge is conserved in closed systems.** In principle, if a negative charge disappeared from your lab bench and reappeared on the Moon, conservation of charge would still hold. However, this never happens. If the total charge you have in your local system on your lab bench is changing, there will be a measurable flow of charge into or out of the system. Again, charges can and do move around, and their effects can and do cancel, but the net charge in your local environment (if closed) is conserved. The last two items are both referred to as the **law of conservation of charge**.

## The Source of Charges: The Structure of the Atom

Once it became clear that all matter was composed of particles that came to be called atoms, it also quickly became clear that the constituents of the atom included both positively charged particles and negatively charged particles. The next question was, what are the physical properties of those electrically charged particles?

The negatively charged particle was the first one to be discovered. In 1897, the English physicist J. J. Thomson was studying what was then known as *cathode rays*. Some years before, the English physicist William Crookes had shown that these "rays" were negatively charged, but his experiments were unable to tell any more than that. (The fact that they carried a negative electric charge was strong evidence that these were not rays at all, but particles.) Thomson prepared a pure beam of these particles and sent them through crossed electric and magnetic fields, and adjusted the various field strengths until the net deflection of the beam was zero. With this experiment, he was able to determine the charge-to-mass ratio of the particle. This ratio showed that the mass of the particle was much smaller than that of any other previously known particle—1837 times smaller, in fact. Eventually, this particle came to be called the **electron**.

Since the atom as a whole is electrically neutral, the next question was to determine how the positive and negative charges are distributed within the atom. Thomson himself imagined that his electrons were embedded within a sort of positively charged paste, smeared out throughout the volume of the atom. However, in 1908, the New Zealand physicist Ernest Rutherford showed that the positive charges of the atom existed within a tiny core—called a nucleus—that took up only a very tiny fraction of the overall volume of the atom, but held over 99% of the mass. (See **Linear Momentum and Collisions (http://cnx.org/content/m58317/latest/)** .) In addition, he showed that the negatively charged electrons perpetually orbited about this nucleus, forming a sort of electrically charged cloud that surrounds the nucleus (**Figure 5.7**). Rutherford concluded that the nucleus was constructed of small, massive particles that he named **protons**.

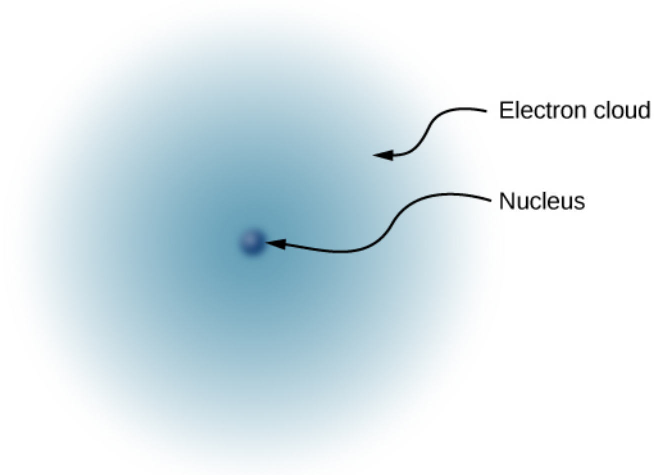



**Figure 5.7** This simplified model of a hydrogen atom shows a positively charged nucleus (consisting, in the case of hydrogen, of a single proton), surrounded by an electron "cloud." The charge of the electron cloud is equal (and opposite in sign) to the charge of the nucleus, but the electron does not have a definite location in space; hence, its representation here is as a cloud. Normal macroscopic amounts of matter contain immense numbers of atoms and molecules, and, hence, even greater numbers of individual negative and positive charges.

Since it was known that different atoms have different masses, and that ordinarily atoms are electrically neutral, it was natural to suppose that different atoms have different numbers of protons in their nucleus, with an equal number of negatively charged electrons orbiting about the positively charged nucleus, thus making the atoms overall electrically neutral. However, it was soon discovered that although the lightest atom, hydrogen, did indeed have a single proton as its nucleus, the next heaviest atom—helium—has twice the number of protons (two), but *four* times the mass of hydrogen.

This mystery was resolved in 1932 by the English physicist James Chadwick, with the discovery of the **neutron**. The neutron is, essentially, an electrically neutral twin of the proton, with no electric charge, but (nearly) identical mass to the proton. The helium nucleus therefore has two neutrons along with its two protons. (Later experiments were to show that although the neutron is electrically neutral overall, it does have an internal charge *structure*. Furthermore, although the masses of the neutron and the proton are *nearly* equal, they aren't exactly equal: The neutron's mass is very slightly larger than the mass of the proton. That slight mass excess turned out to be of great importance. That, however, is a story that will have to wait until our study of modern physics in **Nuclear Physics (http://cnx.org/content/m58606/latest/)** .)

Thus, in 1932, the picture of the atom was of a small, massive nucleus constructed of a combination of protons and neutrons, surrounded by a collection of electrons whose combined motion formed a sort of negatively charged "cloud" around the nucleus (**Figure 5.8**). In an electrically neutral atom, the total negative charge of the collection of electrons is equal to the total positive charge in the nucleus. The very low-mass electrons can be more or less easily removed or added to an atom, changing the net charge on the atom (though without changing its type). An atom that has had the charge altered in this way is called an **ion**. Positive ions have had electrons removed, whereas negative ions have had excess electrons added. We also use this term to describe molecules that are not electrically neutral.

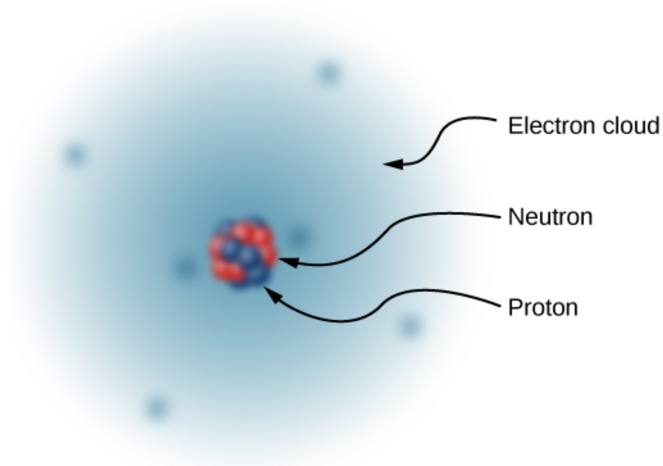



**Figure 5.8** The nucleus of a carbon atom is composed of six protons and six neutrons. As in hydrogen, the surrounding six electrons do not have definite locations and so can be considered to be a sort of cloud surrounding the nucleus.

The story of the atom does not stop there, however. In the latter part of the twentieth century, many more subatomic particles were discovered in the nucleus of the atom: pions, neutrinos, and quarks, among others. With the exception of the photon, none of these particles are directly relevant to the study of electromagnetism, so we defer further discussion of them until the chapter on particle physics (**Particle Physics and Cosmology (http://cnx.org/content/m58767/latest/)** ).

## A Note on Terminology

As noted previously, electric charge is a property that an object can have. This is similar to how an object can have a property that we call mass, a property that we call density, a property that we call temperature, and so on. Technically, we should always say something like, "Suppose we have a particle that carries a charge of $3\ \mu\text{C}$. " However, it is very common to say instead, "Suppose we have a $3\text{-}\mu\text{C}$ charge." Similarly, we often say something like, "Six charges are located at the vertices of a regular hexagon." A charge is not a particle; rather, it is a *property* of a particle. Nevertheless, this terminology is extremely common (and is frequently used in this book, as it is everywhere else). So, keep in the back of your mind what we really mean when we refer to a "charge."

# 5.2 | Conductors, Insulators, and Charging by Induction

## Learning Objectives

By the end of this section, you will be able to:

- Explain what a conductor is
- Explain what an insulator is
- List the differences and similarities between conductors and insulators
- Describe the process of charging by induction

In the preceding section, we said that scientists were able to create electric charge only on nonmetallic materials and never on metals. To understand why this is the case, you have to understand more about the nature and structure of atoms. In this section, we discuss how and why electric charges do—or do not—move through materials (**Figure 5.9**). A more complete description is given in a later chapter.


**Figure 5.9** This power adapter uses metal wires and connectors to conduct electricity from the wall socket to a laptop computer. The conducting wires allow electrons to move freely through the cables, which are shielded by rubber and plastic. These materials act as insulators that don't allow electric charge to escape outward. (credit: modification of work by "Evan-Amos"/Wikimedia Commons)

## Conductors and Insulators

As discussed in the previous section, electrons surround the tiny nucleus in the form of a (comparatively) vast cloud of negative charge. However, this cloud does have a definite structure to it. Let's consider an atom of the most commonly used conductor, copper.

For reasons that will become clear in **Atomic Structure (http://cnx.org/content/m58583/latest/)** , there is an outermost electron that is only loosely bound to the atom's nucleus. It can be easily dislodged; it then moves to a neighboring atom. In a large mass of copper atoms (such as a copper wire or a sheet of copper), these vast numbers of outermost electrons (one per atom) wander from atom to atom, and are the electrons that do the moving when electricity flows. These wandering, or "free," electrons are called **conduction electrons**, and copper is therefore an excellent **conductor** (of electric charge). All conducting elements have a similar arrangement of their electrons, with one or two conduction electrons. This includes most metals.

**Insulators**, in contrast, are made from materials that lack conduction electrons; charge flows only with great difficulty, if at all. Even if excess charge is added to an insulating material, it cannot move, remaining indefinitely in place. This is why insulating materials exhibit the electrical attraction and repulsion forces described earlier, whereas conductors do not; any excess charge placed on a conductor would instantly flow away (due to mutual repulsion from existing charges), leaving no excess charge around to create forces. Charge cannot flow along or through an **insulator**, so its electric forces remain for long periods of time. (Charge will dissipate from an insulator, given enough time.) As it happens, amber, fur, and most semi-precious gems are insulators, as are materials like wood, glass, and plastic.

## Charging by Induction

Let's examine in more detail what happens in a conductor when an electrically charged object is brought close to it. As mentioned, the conduction electrons in the conductor are able to move with nearly complete freedom. As a result, when a charged insulator (such as a positively charged glass rod) is brought close to the conductor, the (total) charge on the insulator exerts an electric force on the conduction electrons. Since the rod is positively charged, the conduction electrons (which themselves are negatively charged) are attracted, flowing toward the insulator to the near side of the conductor (**Figure 5.10**).

Now, the conductor is still overall electrically neutral; the conduction electrons have changed position, but they are still in the conducting material. However, the conductor now has a charge *distribution*; the near end (the portion of the conductor closest to the insulator) now has more negative charge than positive charge, and the reverse is true of the end farthest from the insulator. The relocation of negative charges to the near side of the conductor results in an overall positive charge in the part of the conductor farthest from the insulator. We have thus created an electric charge distribution where one did not exist before. This process is referred to as *inducing polarization*—in this case, polarizing the conductor. The resulting separation of positive and negative charge is called **polarization**, and a material, or even a molecule, that exhibits polarization is said to be polarized. A similar situation occurs with a negatively charged insulator, but the resulting polarization is in the opposite direction.

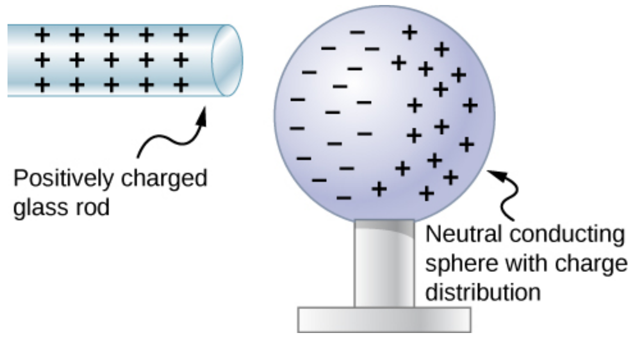



**Figure 5.10** Induced polarization. A positively charged glass rod is brought near the left side of the conducting sphere, attracting negative charge and leaving the other side of the sphere positively charged. Although the sphere is overall still electrically neutral, it now has a charge distribution, so it can exert an electric force on other nearby charges. Furthermore, the distribution is such that it will be attracted to the glass rod.

The result is the formation of what is called an electric **dipole**, from a Latin phrase meaning "two ends." The presence of electric charges on the insulator—and the electric forces they apply to the conduction electrons—creates, or "induces," the dipole in the conductor.

Neutral objects can be attracted to any charged object. The pieces of straw attracted to polished amber are neutral, for example. If you run a plastic comb through your hair, the charged comb can pick up neutral pieces of paper. **Figure 5.11** shows how the polarization of atoms and molecules in neutral objects results in their attraction to a charged object.

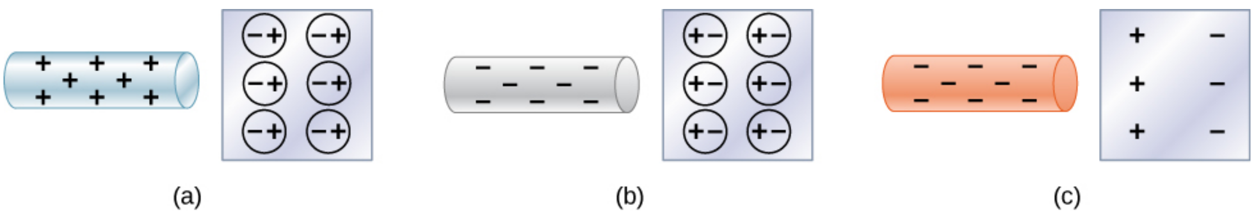



**Figure 5.11** Both positive and negative objects attract a neutral object by polarizing its molecules. (a) A positive object brought near a neutral insulator polarizes its molecules. There is a slight shift in the distribution of the electrons orbiting the molecule, with unlike charges being brought nearer and like charges moved away. Since the electrostatic force decreases with distance, there is a net attraction. (b) A negative object produces the opposite polarization, but again attracts the neutral object. (c) The same effect occurs for a conductor; since the unlike charges are closer, there is a net attraction.

When a charged rod is brought near a neutral substance, an insulator in this case, the distribution of charge in atoms and molecules is shifted slightly. Opposite charge is attracted nearer the external charged rod, while like charge is repelled. Since the electrostatic force decreases with distance, the repulsion of like charges is weaker than the attraction of unlike charges, and so there is a net attraction. Thus, a positively charged glass rod attracts neutral pieces of paper, as will a negatively charged rubber rod. Some molecules, like water, are polar molecules. Polar molecules have a natural or inherent separation of charge, although they are neutral overall. Polar molecules are particularly affected by other charged objects and show greater polarization effects than molecules with naturally uniform charge distributions.

When the two ends of a dipole can be separated, this method of **charging by induction** may be used to create charged objects without transferring charge. In **Figure 5.12**, we see two neutral metal spheres in contact with one another but insulated from the rest of the world. A positively charged rod is brought near one of them, attracting negative charge to that side, leaving the other sphere positively charged.

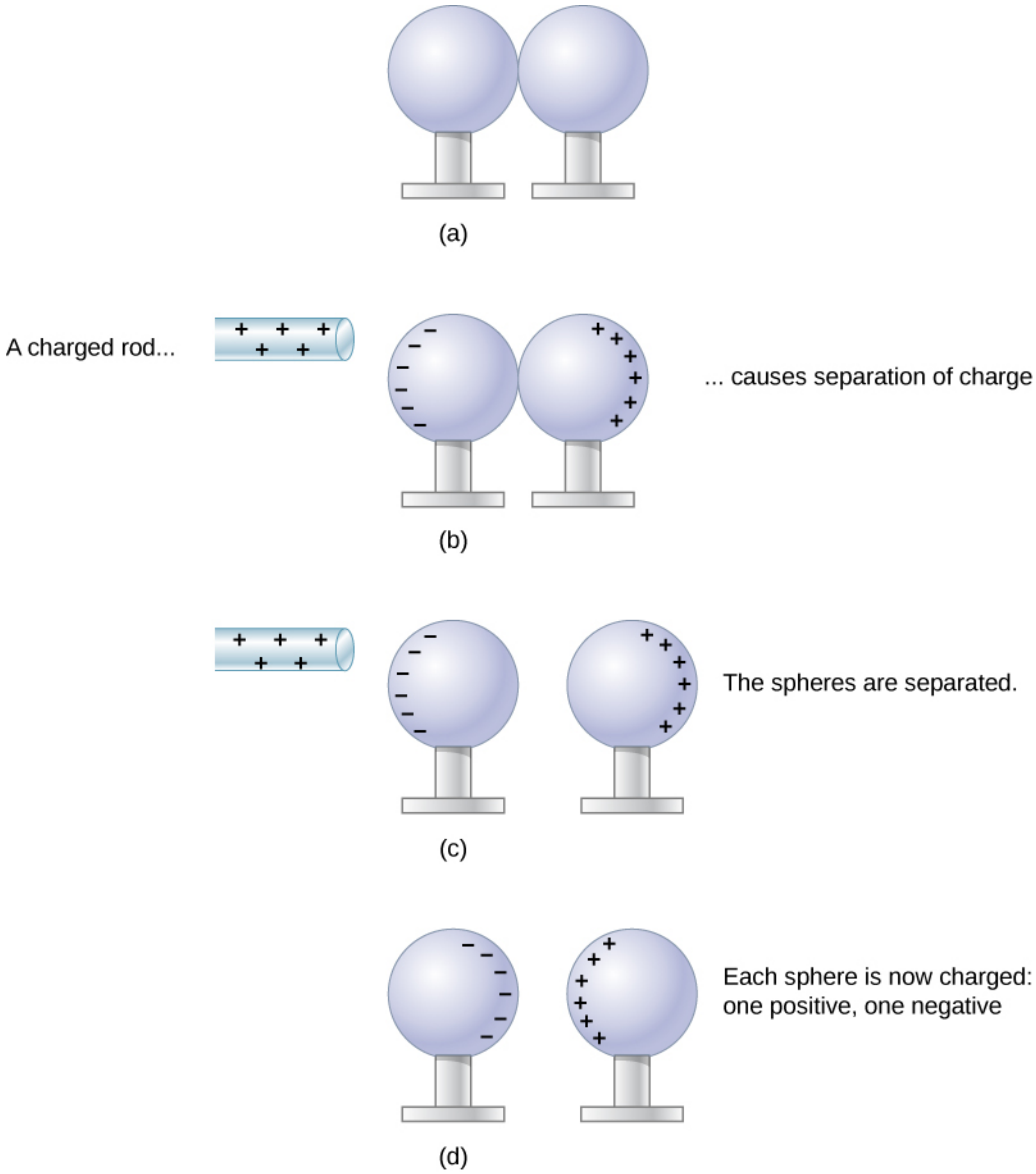



**Figure 5.12** Charging by induction. (a) Two uncharged or neutral metal spheres are in contact with each other but insulated from the rest of the world. (b) A positively charged glass rod is brought near the sphere on the left, attracting negative charge and leaving the other sphere positively charged. (c) The spheres are separated before the rod is removed, thus separating negative and positive charges. (d) The spheres retain net charges after the inducing rod is removed—without ever having been touched by a charged object.

Another method of charging by induction is shown in **Figure 5.13**. The neutral metal sphere is polarized when a charged rod is brought near it. The sphere is then grounded, meaning that a conducting wire is run from the sphere to the ground. Since Earth is large and most of the ground is a good conductor, it can supply or accept excess charge easily. In this case, electrons are attracted to the sphere through a wire called the ground wire, because it supplies a conducting path to the ground. The ground connection is broken before the charged rod is removed, leaving the sphere with an excess charge opposite to that of the rod. Again, an opposite charge is achieved when charging by induction, and the charged rod loses none of its excess charge.

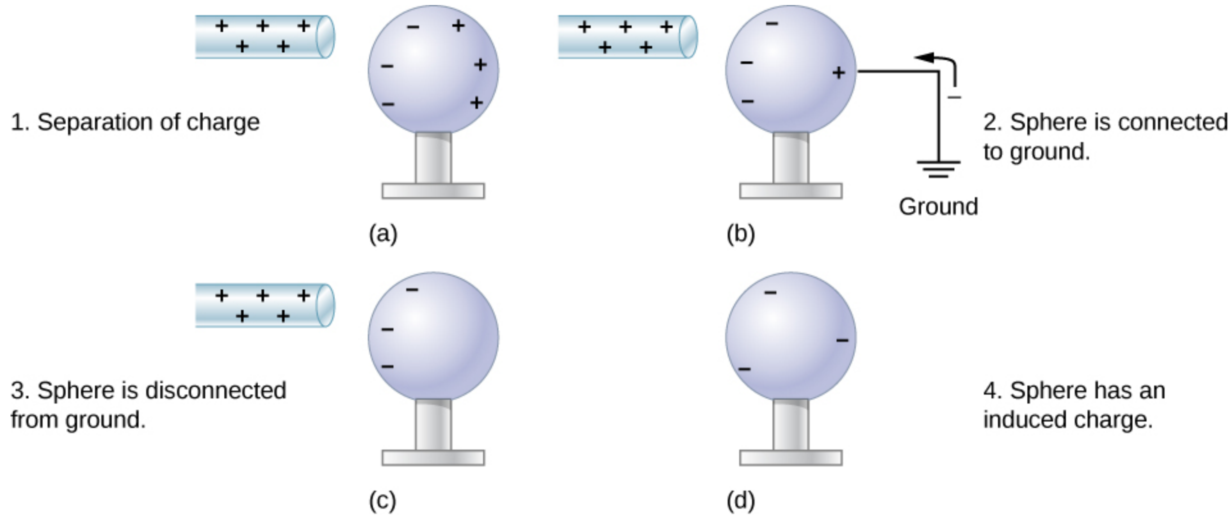



**Figure 5.13** Charging by induction using a ground connection. (a) A positively charged rod is brought near a neutral metal sphere, polarizing it. (b) The sphere is grounded, allowing electrons to be attracted from Earth's ample supply. (c) The ground connection is broken. (d) The positive rod is removed, leaving the sphere with an induced negative charge.

# 5.3 | Coulomb's Law

## Learning Objectives

By the end of this section, you will be able to:

- Describe the electric force, both qualitatively and quantitatively
- Calculate the force that charges exert on each other
- Determine the direction of the electric force for different source charges
- Correctly describe and apply the superposition principle for multiple source charges

Experiments with electric charges have shown that if two objects each have electric charge, then they exert an electric force on each other. The magnitude of the force is linearly proportional to the net charge on each object and inversely proportional to the square of the distance between them. (Interestingly, the force does not depend on the mass of the objects.) The direction of the force vector is along the imaginary line joining the two objects and is dictated by the signs of the charges involved.

Let

- $q_1, q_2 =$ the net electric charges of the two objects;
- $\vec{\mathbf{r}}_{12} =$ the vector displacement from $q_1$ to $q_2$.

The electric force $\vec{\mathbf{F}}$ on one of the charges is proportional to the magnitude of its own charge and the magnitude of the other charge, and is inversely proportional to the square of the distance between them:

$$F \propto \frac{q_1 q_2}{r_{12}^2}.$$

This proportionality becomes an equality with the introduction of a proportionality constant. For reasons that will become clear in a later chapter, the proportionality constant that we use is actually a collection of constants. (We discuss this constant shortly.)

### Coulomb's Law

The magnitude of the electric force (or **Coulomb force**) between two electrically charged particles is equal to

$$\vec{\mathbf{F}}_{12}(r) = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r_{12}^2}\hat{\mathbf{r}}_{12} \tag{5.1}$$

We use absolute value signs around the product $q_1 q_2$ because one of the charges may be negative, but the magnitude of the force is always positive. The direction of the force vector depends on the sign of the charges. If the charges are the same, the force points away from the other charge. If the charges have different signs, the force points toward the other charge(**Figure 5.14**).

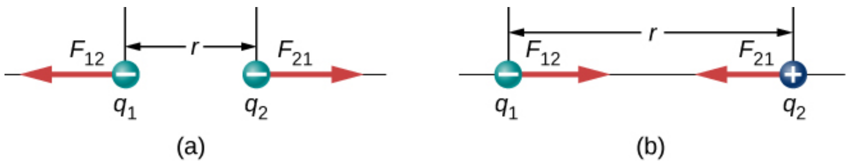



**Figure 5.14** The electrostatic force $\vec{\mathbf{F}}$ between point charges $q_1$ and $q_2$ separated by a distance *r* is given by Coulomb's law. Note that Newton's third law (every force exerted creates an equal and opposite force) applies as usual—the force on $q_1$ is equal in magnitude and opposite in direction to the force it exerts on $q_2$. (a) Like charges; (b) unlike charges.

It is important to note that the electric force is not constant; it is a function of the separation distance between the two charges. If either the test charge or the source charge (or both) move, then $\vec{\mathbf{r}}$ changes, and therefore so does the force. An immediate consequence of this is that direct application of Newton's laws with this force can be mathematically difficult, depending on the specific problem at hand. It can (usually) be done, but we almost always look for easier methods of calculating whatever physical quantity we are interested in. (Conservation of energy is the most common choice.)

Finally, the new constant $\varepsilon_0$ in Coulomb's law is called the *permittivity of free space*, or (better) the **permittivity of vacuum**. It has a very important physical meaning that we will discuss in a later chapter; for now, it is simply an empirical proportionality constant. Its numerical value (to three significant figures) turns out to be

$$\varepsilon_0 = 8.85 \times 10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N \cdot m}^2}.$$

These units are required to give the force in Coulomb's law the correct units of newtons. Note that in Coulomb's law, the permittivity of vacuum is only part of the proportionality constant. For convenience, we often define a Coulomb's constant:

$$k_e = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^{9}\,\frac{\mathrm{N \cdot m}^2}{\mathrm{C}^2}.$$

## Example 5.1

### The Force on the Electron in Hydrogen

A hydrogen atom consists of a single proton and a single electron. The proton has a charge of $+e$ and the electron has $-e$. In the “ground state” of the atom, the electron orbits the proton at most probable distance of $5.29 \times 10^{-11}$ m (**Figure 5.15**). Calculate the electric force on the electron due to the proton.

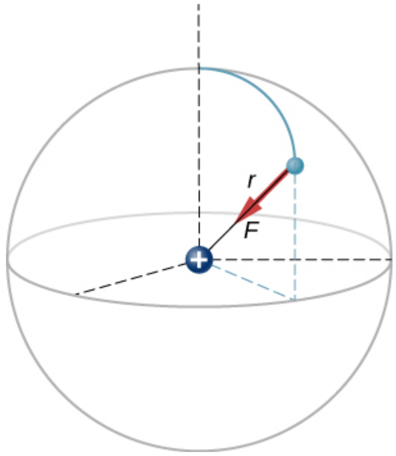



**Figure 5.15** A schematic depiction of a hydrogen atom, showing the force on the electron. This depiction is only to enable us to calculate the force; the hydrogen atom does not really look like this. Recall **Figure 5.7**.

### Strategy

For the purposes of this example, we are treating the electron and proton as two point particles, each with an electric charge, and we are told the distance between them; we are asked to calculate the force on the electron. We thus use Coulomb's law.

### Solution

Our two charges and the distance between them are,

$$\begin{aligned} q_1 &= +e = +1.602 \times 10^{-19}\ \text{C} \\ q_2 &= -e = -1.602 \times 10^{-19}\ \text{C} \\ r &= 5.29 \times 10^{-11}\ \text{m}. \end{aligned}$$

The magnitude of the force on the electron is

$$F = \frac{1}{4\pi\epsilon_0}\frac{|e|^2}{r^2} = \frac{1}{4\pi\left(8.85 \times 10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)}\frac{\left(1.602 \times 10^{-19}\ \text{C}\right)^2}{\left(5.29 \times 10^{-11}\ \text{m}\right)^2} = 8.25 \times 10^{-8}\ \text{N}.$$

As for the direction, since the charges on the two particles are opposite, the force is attractive; the force on the electron points radially directly toward the proton, everywhere in the electron's orbit. The force is thus expressed as

$$\vec{\mathbf{F}} = \left(8.25 \times 10^{-8}\ \text{N}\right)\hat{\mathbf{r}}.$$

### Significance

This is a three-dimensional system, so the electron (and therefore the force on it) can be anywhere in an imaginary spherical shell around the proton. In this "classical" model of the hydrogen atom, the electrostatic force on the electron points in the inward centripetal direction, thus maintaining the electron's orbit. But note that the quantum mechanical model of hydrogen (discussed in **Quantum Mechanics (http://cnx.org/content/m58573/latest/)** ) is utterly different.

**5.1 Check Your Understanding** What would be different if the electron also had a positive charge?

## Multiple Source Charges

The analysis that we have done for two particles can be extended to an arbitrary number of particles; we simply repeat the analysis, two charges at a time. Specifically, we ask the question: Given $N$ charges (which we refer to as source charge), what is the net electric force that they exert on some other point charge (which we call the test charge)? Note that we use these terms because we can think of the test charge being used to test the strength of the force provided by the source charges.

Like all forces that we have seen up to now, the net electric force on our test charge is simply the vector sum of each individual electric force exerted on it by each of the individual test charges. Thus, we can calculate the net force on the test charge $Q$ by calculating the force on it from each source charge, taken one at a time, and then adding all those forces together (as vectors). This ability to simply add up individual forces in this way is referred to as the **principle of superposition**, and is one of the more important features of the electric force. In mathematical form, this becomes

$$\vec{\mathbf{F}}(r) = \frac{1}{4\pi\varepsilon_0} Q \sum_{i=1}^{N} \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i. \tag{5.2}$$

In this expression, $Q$ represents the charge of the particle that is experiencing the electric force $\vec{\mathbf{F}}$, and is located at $\vec{\mathbf{r}}$ from the origin; the $q_i$'s are the $N$ source charges, and the vectors $\vec{\mathbf{r}}_i = r_i \hat{\mathbf{r}}_i$ are the displacements from the position of the $i$th charge to the position of $Q$. Each of the $N$ unit vectors points directly from its associated source charge toward the test charge. All of this is depicted in **Figure 5.16**. Please note that there is no physical difference between $Q$ and $q_i$; the difference in labels is merely to allow clear discussion, with $Q$ being the charge we are determining the force on.

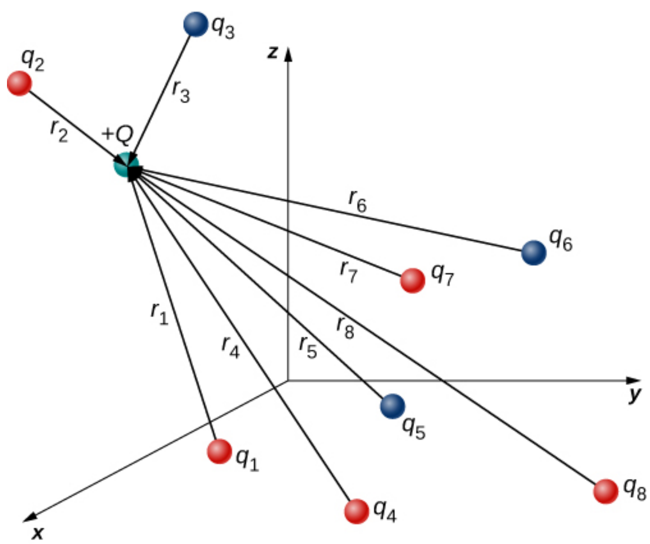



**Figure 5.16** The eight source charges each apply a force on the single test charge $Q$. Each force can be calculated independently of the other seven forces. This is the essence of the superposition principle.

(Note that the force vector $\vec{\mathbf{F}}_i$ does not necessarily point in the same direction as the unit vector $\hat{\mathbf{r}}_i$; it may point in the opposite direction, $-\hat{\mathbf{r}}_i$. The signs of the source charge and test charge determine the direction of the force on the test charge.)

There is a complication, however. Just as the source charges each exert a force on the test charge, so too (by Newton's third law) does the test charge exert an equal and opposite force on each of the source charges. As a consequence, each source charge would change position. However, by **Equation 5.2**, the force on the test charge is a function of position; thus, as the positions of the source charges change, the net force on the test charge necessarily changes, which changes the force, which again changes the positions. Thus, the entire mathematical analysis quickly becomes intractable. Later, we will learn techniques for handling this situation, but for now, we make the simplifying assumption that the source charges are fixed in place somehow, so that their positions are constant in time. (The test charge is allowed to move.) With this restriction in place, the analysis of charges is known as **electrostatics**, where "statics" refers to the constant (that is, static) positions of the source charges and the force is referred to as an **electrostatic force**.

## Example 5.2

### The Net Force from Two Source Charges

Three different, small charged objects are placed as shown in Figure 5.17. The charges $q_1$ and $q_3$ are fixed in place; $q_2$ is free to move. Given $q_1 = 2e$, $q_2 = -3e$, and $q_3 = -5e$, and that $d = 2.0 \times 10^{-7}$ m, what is the net force on the middle charge $q_2$?

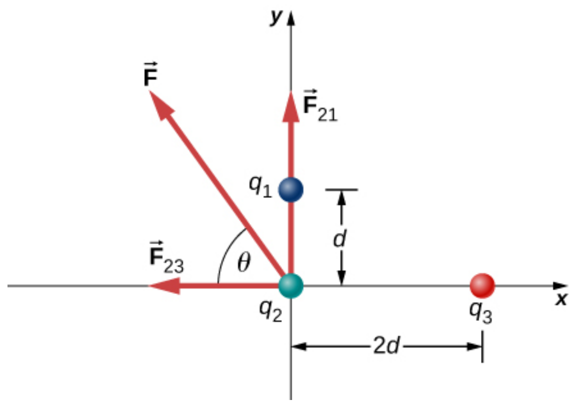



Figure 5.17 Source charges $q_1$ and $q_3$ each apply a force on $q_2$.

### Strategy

We use Coulomb's law again. The way the question is phrased indicates that $q_2$ is our test charge, so that $q_1$ and $q_3$ are source charges. The principle of superposition says that the force on $q_2$ from each of the other charges is unaffected by the presence of the other charge. Therefore, we write down the force on $q_2$ from each and add them together as vectors.

### Solution

We have two source charges $(q_1$ and $q_3)$, a test charge $(q_2)$, distances $(r_{21}$ and $r_{23})$, and we are asked to find a force. This calls for Coulomb's law and superposition of forces. There are two forces:

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_{21} + \vec{\mathbf{F}}_{23} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_2 q_1}{r_{21}^2}\hat{\mathbf{j}} + \left(-\frac{q_2 q_3}{r_{23}^2}\hat{\mathbf{i}}\right)\right].$$

We can't add these forces directly because they don't point in the same direction: $\vec{\mathbf{F}}_{12}$ points only in the −*x*-direction, while $\vec{\mathbf{F}}_{13}$ points only in the +*y*-direction. The net force is obtained from applying the Pythagorean theorem to its *x*- and *y*-components:

$$F = \sqrt{F_x^2 + F_y^2}$$

where

$$\begin{aligned} F_x &= -F_{23} = -\frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{r_{23}^2} \\ &= -\left(8.99\times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(4.806\times 10^{-19}\ \text{C})(8.01\times 10^{-19}\ \text{C})}{(4.00\times 10^{-7}\ \text{m})^2} \\ &= -2.16\times 10^{-14}\ \text{N} \end{aligned}$$

and

$$\begin{aligned} F_y &= F_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_1}{r_{21}^2} \\ &= \left(8.99\times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(4.806\times 10^{-19}\ \text{C})(3.204\times 10^{-19}\ \text{C})}{(2.00\times 10^{-7}\ \text{m})^2} \\ &= 3.46\times 10^{-14}\ \text{N}. \end{aligned}$$

We find that

$$F = \sqrt{F_x^2 + F_y^2} = 4.08\times 10^{-14}\ \text{N}$$

at an angle of

$$\phi = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{3.46\times 10^{-14}\ \text{N}}{-2.16\times 10^{-14}\ \text{N}}\right) = -58°,$$

that is, $58°$ above the $-x$-axis, as shown in the diagram.

### Significance

Notice that when we substituted the numerical values of the charges, we did not include the negative sign of either $q_2$ or $q_3$. Recall that negative signs on vector quantities indicate a reversal of direction of the vector in question. But for electric forces, the direction of the force is determined by the types (signs) of both interacting charges; we determine the force directions by considering whether the signs of the two charges are the same or are opposite. If you also include negative signs from negative charges when you substitute numbers, you run the risk of mathematically reversing the direction of the force you are calculating. Thus, the safest thing to do is to calculate just the magnitude of the force, using the absolute values of the charges, and determine the directions physically.

It's also worth noting that the only new concept in this example is how to calculate the electric forces; everything else (getting the net force from its components, breaking the forces into their components, finding the direction of the net force) is the same as force problems you have done earlier.

**5.2 Check Your Understanding** What would be different if $q_1$ were negative?

# 5.4 | Electric Field

## Learning Objectives

By the end of this section, you will be able to:

- Explain the purpose of the electric field concept
- Describe the properties of the electric field
- Calculate the field of a collection of source charges of either sign

As we showed in the preceding section, the net electric force on a test charge is the vector sum of all the electric forces acting on it, from all of the various source charges, located at their various positions. But what if we use a different test charge, one with a different magnitude, or sign, or both? Or suppose we have a dozen different test charges we wish to try at the same location? We would have to calculate the sum of the forces from scratch. Fortunately, it is possible to define a quantity, called the **electric field**, which is independent of the test charge. It only depends on the configuration of the source charges, and once found, allows us to calculate the force on any test charge.

## Defining a Field

Suppose we have $N$ source charges $q_1, q_2, q_3, \ldots, q_N$ located at positions $\vec{\mathbf{r}}_1, \vec{\mathbf{r}}_2, \vec{\mathbf{r}}_3, \ldots, \vec{\mathbf{r}}_N$, applying $N$ electrostatic forces on a test charge $Q$. The net force on $Q$ is (see **Equation 5.2**)

$$\begin{aligned}\vec{\mathbf{F}} &= \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_3 + \cdots + \vec{\mathbf{F}}_N \\ &= \frac{1}{4\pi\varepsilon_0}\left(\frac{Qq_1}{r_1^2}\hat{\mathbf{r}}_1 + \frac{Qq_2}{r_2^2}\hat{\mathbf{r}}_2 + \frac{Qq_3}{r_3^2}\hat{\mathbf{r}}_3 + \cdots + \frac{Qq_N}{r_1^2}\hat{\mathbf{r}}_N\right) \\ &= Q\left[\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1^2}\hat{\mathbf{r}}_1 + \frac{q_2}{r_2^2}\hat{\mathbf{r}}_2 + \frac{q_3}{r_3^2}\hat{\mathbf{r}}_3 + \cdots + \frac{q_N}{r_1^2}\hat{\mathbf{r}}_N\right)\right].\end{aligned}$$

We can rewrite this as

$$\vec{\mathbf{F}} = Q\vec{\mathbf{E}} \tag{5.3}$$

where

$$\vec{\mathbf{E}} \equiv \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1^2}\hat{\mathbf{r}}_1 + \frac{q_2}{r_2^2}\hat{\mathbf{r}}_2 + \frac{q_3}{r_3^2}\hat{\mathbf{r}}_3 + \cdots + \frac{q_N}{r_1^2}\hat{\mathbf{r}}_N\right)$$

or, more compactly,

$$\vec{\mathbf{E}}(P) \equiv \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_i^2}\hat{\mathbf{r}}_i. \tag{5.4}$$

This expression is called the electric field at position $P = P(x, y, z)$ of the $N$ source charges. Here, $P$ is the location of the point in space where you are calculating the field and is relative to the positions $\vec{\mathbf{r}}_i$ of the source charges (**Figure 5.18**). Note that we have to impose a coordinate system to solve actual problems.

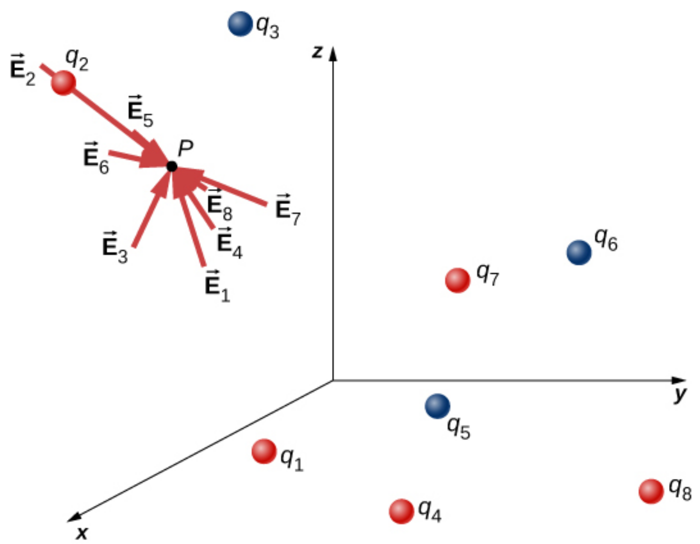



**Figure 5.18** Each of these eight source charges creates its own electric field at every point in space; shown here are the field vectors at an arbitrary point *P*. Like the electric force, the net electric field obeys the superposition principle.

Notice that the calculation of the electric field makes no reference to the test charge. Thus, the physically useful approach is to calculate the electric field and then use it to calculate the force on some test charge later, if needed. Different test charges experience different forces **Equation 5.3**, but it is the same electric field **Equation 5.4**. That being said, recall that there is no fundamental difference between a test charge and a source charge; these are merely convenient labels for the system of interest. Any charge produces an electric field; however, just as Earth's orbit is not affected by Earth's own gravity, a charge is not subject to a force due to the electric field it generates. Charges are only subject to forces from the electric fields of other charges.

In this respect, the electric field $\vec{\mathbf{E}}$ of a point charge is similar to the gravitational field $\vec{\mathbf{g}}$ of Earth; once we have calculated the gravitational field at some point in space, we can use it any time we want to calculate the resulting force on any mass we choose to place at that point. In fact, this is exactly what we do when we say the gravitational field of Earth (near Earth's surface) has a value of $9.81 \text{ m/s}^2$, and then we calculate the resulting force (i.e., weight) on different masses. Also, the general expression for calculating $\vec{\mathbf{g}}$ at arbitrary distances from the center of Earth (i.e., not just near Earth's surface) is very similar to the expression for $\vec{\mathbf{E}}$ : $\vec{\mathbf{g}} = G\frac{M}{r^2}\hat{\mathbf{r}}$, where *G* is a proportionality constant, playing the same role for $\vec{\mathbf{g}}$ as $\frac{1}{4\pi\varepsilon_0}$ does for $\vec{\mathbf{E}}$. The value of $\vec{\mathbf{g}}$ is calculated once and is then used in an endless number of problems.

To push the analogy further, notice the units of the electric field: From $F = QE$, the units of *E* are newtons per coulomb, N/C, that is, the electric field applies a force on each unit charge. Now notice the units of *g*: From $w = mg$, the units of *g* are newtons per kilogram, N/kg, that is, the gravitational field applies a force on each unit mass. We could say that the gravitational field of Earth, near Earth's surface, has a value of 9.81 N/kg.

## The Meaning of "Field"

Recall from your studies of gravity that the word "field" in this context has a precise meaning. A field, in physics, is a physical quantity whose value depends on (is a function of) position, relative to the source of the field. In the case of the electric field, **Equation 5.4** shows that the value of $\vec{\mathbf{E}}$ (both the magnitude and the direction) depends on where in space the point *P* is located, measured from the locations $\vec{\mathbf{r}}_i$ of the source charges $q_i$.

In addition, since the electric field is a vector quantity, the electric field is referred to as a *vector field*. (The gravitational field is also a vector field.) In contrast, a field that has only a magnitude at every point is a *scalar field*. The temperature in a room is an example of a scalar field. It is a field because the temperature, in general, is different at different locations in the room, and it is a scalar field because temperature is a scalar quantity.

Also, as you did with the gravitational field of an object with mass, you should picture the electric field of a charge-bearing object (the source charge) as a continuous, immaterial substance that surrounds the source charge, filling all of space—in principle, to $\pm\infty$ in all directions. The field exists at every physical point in space. To put it another way, the electric charge on an object alters the space around the charged object in such a way that all other electrically charged objects in space experience an electric force as a result of being in that field. The electric field, then, is the mechanism by which the electric properties of the source charge are transmitted to and through the rest of the universe. (Again, the range of the electric force is infinite.)

We will see in subsequent chapters that the speed at which electrical phenomena travel is the same as the speed of light. There is a deep connection between the electric field and light.

## Superposition

Yet another experimental fact about the field is that it obeys the superposition principle. In this context, that means that we can (in principle) calculate the total electric field of many source charges by calculating the electric field of only $q_1$ at position *P*, then calculate the field of $q_2$ at *P*, while—and this is the crucial idea—ignoring the field of, and indeed even the existence of, $q_1$. We can repeat this process, calculating the field of each individual source charge, independently of the existence of any of the other charges. The total electric field, then, is the vector sum of all these fields. That, in essence, is what **Equation 5.4** says.

In the next section, we describe how to determine the shape of an electric field of a source charge distribution and how to sketch it.

## The Direction of the Field

**Equation 5.4** enables us to determine the magnitude of the electric field, but we need the direction also. We use the convention that the direction of any electric field vector is the same as the direction of the electric force vector that the field would apply to a positive test charge placed in that field. Such a charge would be repelled by positive source charges (the force on it would point away from the positive source charge) but attracted to negative charges (the force points toward the negative source).

### Direction of the Electric Field

By convention, all electric fields $\vec{\mathbf{E}}$ point away from positive source charges and point toward negative source charges.

Add charges to the **Electric Field of Dreams (https://openstaxcollege.org/l/21elefiedream)** and see how they react to the electric field. Turn on a background electric field and adjust the direction and magnitude.

## Example 5.3

### The *E*-field of an Atom

In an ionized helium atom, the most probable distance between the nucleus and the electron is $r = 26.5 \times 10^{-12}\ \text{m}$. What is the electric field due to the nucleus at the location of the electron?

### Strategy

Note that although the electron is mentioned, it is not used in any calculation. The problem asks for an electric field, not a force; hence, there is only one charge involved, and the problem specifically asks for the field due to the nucleus. Thus, the electron is a red herring; only its distance matters. Also, since the distance between the two protons in the nucleus is much, much smaller than the distance of the electron from the nucleus, we can treat the two protons as a single charge +2*e* (**Figure 5.19**).

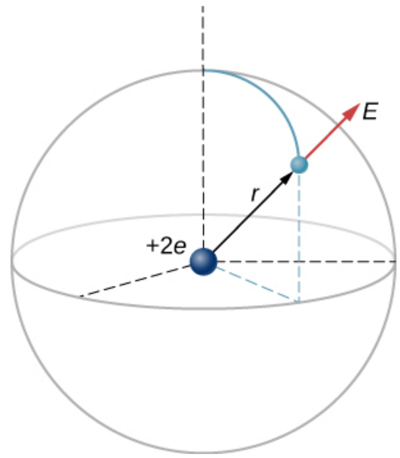



**Figure 5.19** A schematic representation of a helium atom. Again, helium physically looks nothing like this, but this sort of diagram is helpful for calculating the electric field of the nucleus.

### Solution

The electric field is calculated by

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i.$$

Since there is only one source charge (the nucleus), this expression simplifies to

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}.$$

Here $q = 2e = 2\left(1.6 \times 10^{-19} \text{ C}\right)$ (since there are two protons) and $r$ is given; substituting gives

$$\vec{\mathbf{E}} = \frac{1}{4\pi\left(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}\right)} \frac{2\left(1.6 \times 10^{-19} \text{ C}\right)}{\left(26.5 \times 10^{-12} \text{ m}\right)^2} \hat{\mathbf{r}} = 4.1 \times 10^{12} \frac{\text{N}}{\text{C}} \hat{\mathbf{r}}.$$

The direction of $\vec{\mathbf{E}}$ is radially away from the nucleus in all directions. Why? Because a positive test charge placed in this field would accelerate radially away from the nucleus (since it is also positively charged), and again, the convention is that the direction of the electric field vector is defined in terms of the direction of the force it would apply to positive test charges.

## Example 5.4

### The *E*-Field above Two Equal Charges

(a) Find the electric field (magnitude and direction) a distance $z$ above the midpoint between two equal charges $+q$ that are a distance $d$ apart (**Figure 5.20**). Check that your result is consistent with what you'd expect when $z \gg d$.

(b) The same as part (a), only this time make the right-hand charge $-q$ instead of $+q$.

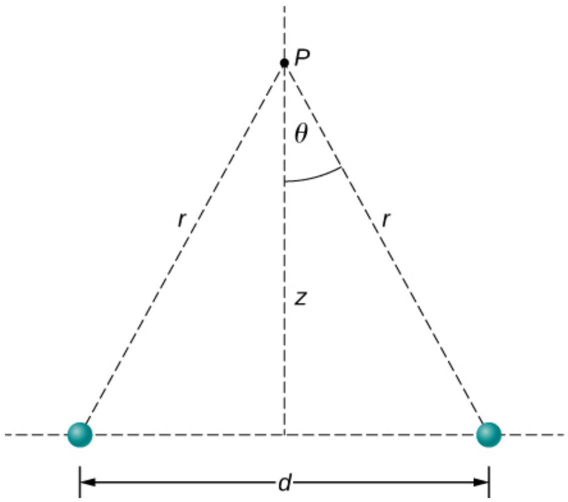



**Figure 5.20** Finding the field of two identical source charges at the point $P$. Due to the symmetry, the net field at $P$ is entirely vertical. (Notice that this is *not* true away from the midline between the charges.)

### Strategy

We add the two fields as vectors, per **Equation 5.4**. Notice that the system (and therefore the field) is symmetrical about the vertical axis; as a result, the horizontal components of the field vectors cancel. This simplifies the math. Also, we take care to express our final answer in terms of only quantities that are given in the original statement of the problem: $q$, $z$, $d$, and constants $(\pi,\ \varepsilon_0)$.

### Solution

a. By symmetry, the horizontal ($x$)-components of $\vec{\mathbf{E}}$ cancel (**Figure 5.21**); $E_x = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\sin\theta - \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\sin\theta = 0\,.$

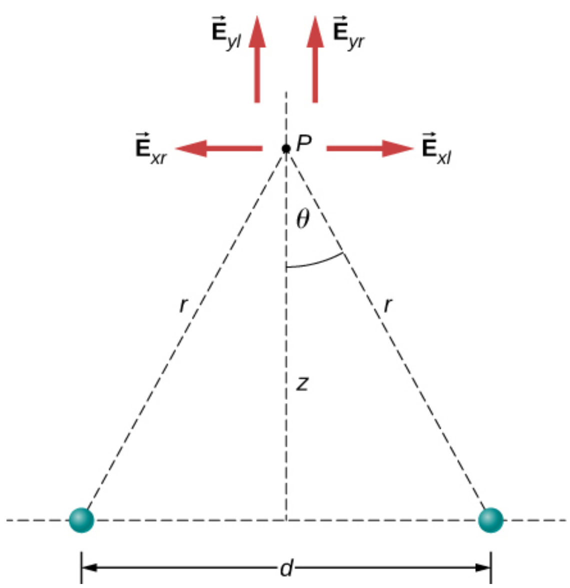



**Figure 5.21** Note that the horizontal components of the electric fields from the two charges cancel each other out, while the vertical components add together.

The vertical ($z$)-component is given by

$$E_z = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\cos\theta + \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}\cos\theta.$$

Since none of the other components survive, this is the entire electric field, and it points in the $\hat{\mathbf{k}}$ direction. Notice that this calculation uses the principle of **superposition**; we calculate the fields of the two charges independently and then add them together.

What we want to do now is replace the quantities in this expression that we don't know (such as $r$), or can't easily measure (such as $\cos\theta$) with quantities that we do know, or can measure. In this case, by geometry,

$$r^2 = z^2 + \left(\frac{d}{2}\right)^2$$

and

$$\cos\theta = \frac{z}{r} = \frac{z}{\left[z^2 + \left(\frac{d}{2}\right)^2\right]^{1/2}}.$$

Thus, substituting,

$$\vec{\mathbf{E}}(z) = \frac{1}{4\pi\varepsilon_0}\frac{2q}{\left[z^2 + \left(\frac{d}{2}\right)^2\right]}\frac{z}{\left[z^2 + \left(\frac{d}{2}\right)^2\right]^{1/2}}\hat{\mathbf{k}}.$$

Simplifying, the desired answer is

$$\vec{\mathbf{E}}(z) = \frac{1}{4\pi\varepsilon_0}\frac{2qz}{\left[z^2 + \left(\frac{d}{2}\right)^2\right]^{3/2}}\hat{\mathbf{k}}. \tag{5.5}$$

b. If the source charges are equal and opposite, the vertical components cancel because $E_z = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\cos\theta - \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\cos\theta = 0$

and we get, for the horizontal component of $\vec{\mathbf{E}}$,

$$\begin{aligned}\vec{\mathbf{E}}(z) &= \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\sin\theta\,\hat{\mathbf{i}} - \frac{1}{4\pi\varepsilon_0}\frac{-q}{r^2}\sin\theta\,\hat{\mathbf{i}} \\ &= \frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}\sin\theta\,\hat{\mathbf{i}} \\ &= \frac{1}{4\pi\varepsilon_0}\frac{2q}{\left[z^2 + \left(\frac{d}{2}\right)^2\right]}\frac{\left(\frac{d}{2}\right)}{\left[z^2 + \left(\frac{d}{2}\right)^2\right]^{1/2}}\hat{\mathbf{i}}.\end{aligned}$$

This becomes

$$\vec{\mathbf{E}}(z) = \frac{1}{4\pi\varepsilon_0}\frac{qd}{\left[z^2 + \left(\frac{d}{2}\right)^2\right]^{3/2}}\hat{\mathbf{i}}. \tag{5.6}$$

### Significance

It is a very common and very useful technique in physics to check whether your answer is reasonable by evaluating it at extreme cases. In this example, we should evaluate the field expressions for the cases $d = 0$, $z \gg d$, and $z \to \infty$, and confirm that the resulting expressions match our physical expectations. Let's do so:

Let's start with **Equation 5.5**, the field of two identical charges. From far away (i.e., $z \gg d$), the two source charges should "merge" and we should then "see" the field of just one charge, of size $2q$. So, let $z \gg d$; then we can neglect $d^2$ in **Equation 5.5** to obtain

$$\begin{aligned}\lim_{d \to 0} \vec{\mathbf{E}} &= \frac{1}{4\pi\varepsilon_0} \frac{2qz}{\left[z^2\right]^{3/2}} \hat{\mathbf{k}} \\ &= \frac{1}{4\pi\varepsilon_0} \frac{2qz}{z^3} \hat{\mathbf{k}} \\ &= \frac{1}{4\pi\varepsilon_0} \frac{(2q)}{z^2} \hat{\mathbf{k}},\end{aligned}$$

which is the correct expression for a field at a distance $z$ away from a charge $2q$.

Next, we consider the field of equal and opposite charges, **Equation 5.6**. It can be shown (via a Taylor expansion) that for $d \ll z \ll \infty$, this becomes

$$\vec{\mathbf{E}}(z) = \frac{1}{4\pi\varepsilon_0} \frac{qd}{z^3} \hat{\mathbf{i}}, \tag{5.7}$$

which is the field of a dipole, a system that we will study in more detail later. (Note that the units of $\vec{\mathbf{E}}$ are still correct in this expression, since the units of $d$ in the numerator cancel the unit of the "extra" $z$ in the denominator.) If $z$ is *very* large $(z \to \infty)$, then $E \to 0$, as it should; the two charges "merge" and so cancel out.

**5.3 Check Your Understanding** What is the electric field due to a single point particle?

Try this **simulation of electric field hockey (https://openstaxcollege.org/l/21elefielhocke)** to get the charge in the goal by placing other charges on the field.

# 5.5 | Calculating Electric Fields of Charge Distributions

## Learning Objectives

By the end of this section, you will be able to:

- Explain what a continuous source charge distribution is and how it is related to the concept of quantization of charge
- Describe line charges, surface charges, and volume charges
- Calculate the field of a continuous source charge distribution of either sign

The charge distributions we have seen so far have been discrete: made up of individual point particles. This is in contrast with a **continuous charge distribution**, which has at least one nonzero dimension. If a charge distribution is continuous rather than discrete, we can generalize the definition of the electric field. We simply divide the charge into infinitesimal pieces and treat each piece as a point charge.

Note that because charge is quantized, there is no such thing as a "truly" continuous charge distribution. However, in most practical cases, the total charge creating the field involves such a huge number of discrete charges that we can safely ignore the discrete nature of the charge and consider it to be continuous. This is exactly the kind of approximation we make when we deal with a bucket of water as a continuous fluid, rather than a collection of $H_2O$ molecules.

Our first step is to define a charge density for a charge distribution along a line, across a surface, or within a volume, as shown in **Figure 5.22**.

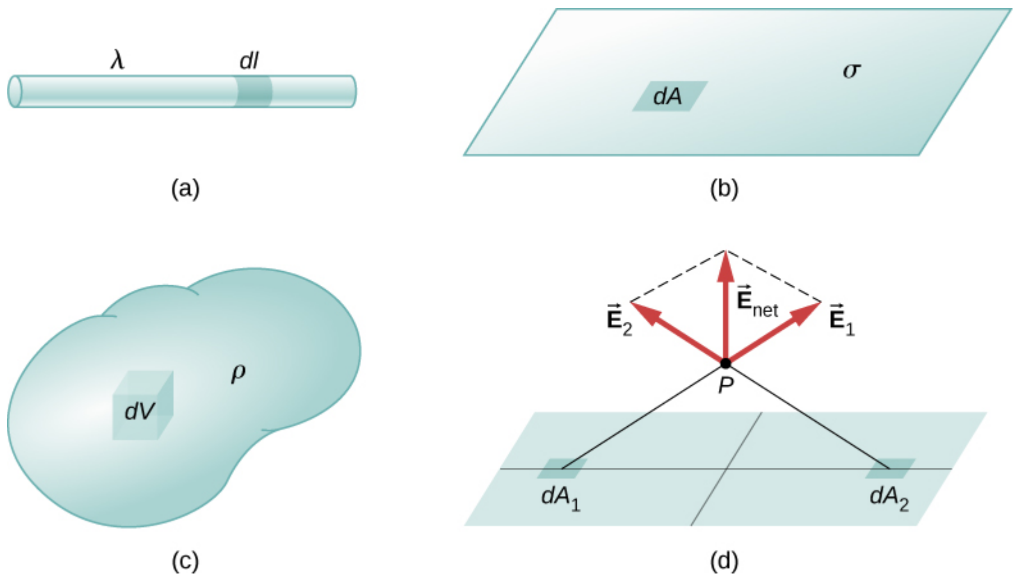



**Figure 5.22** The configuration of charge differential elements for a (a) line charge, (b) sheet of charge, and (c) a volume of charge. Also note that (d) some of the components of the total electric field cancel out, with the remainder resulting in a net electric field.

Definitions of charge density:

- $\lambda \equiv$ charge per unit length ( **linear charge density**); units are coulombs per meter (C/m)
- $\sigma \equiv$ charge per unit area ( **surface charge density**); units are coulombs per square meter $(\mathrm{C/m}^2)$
- $\rho \equiv$ charge per unit volume ( **volume charge density**); units are coulombs per cubic meter $(\mathrm{C/m}^3)$

Then, for a line charge, a surface charge, and a volume charge, the summation in **Equation 5.4** becomes an integral and $q_i$ is replaced by $dq = \lambda dl$ , $\sigma dA$ , or $\rho dV$ , respectively:

$$\text{Point charge:} \quad \vec{\mathbf{E}}(P) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\left(\frac{q_i}{r^2}\right)\hat{\mathbf{r}} \tag{5.8}$$

$$\text{Line charge:} \quad \vec{\mathbf{E}}(P) = \frac{1}{4\pi\varepsilon_0}\int_{\text{line}}\left(\frac{\lambda dl}{r^2}\right)\hat{\mathbf{r}} \tag{5.9}$$

$$\text{Surface charge:} \quad \vec{\mathbf{E}}(P) = \frac{1}{4\pi\varepsilon_0}\int_{\text{surface}}\left(\frac{\sigma dA}{r^2}\right)\hat{\mathbf{r}} \tag{5.10}$$

$$\text{Volume charge:} \quad \vec{\mathbf{E}}(P) = \frac{1}{4\pi\varepsilon_0}\int_{\text{volume}}\left(\frac{\rho dV}{r^2}\right)\hat{\mathbf{r}} \tag{5.11}$$

The integrals are generalizations of the expression for the field of a point charge. They implicitly include and assume the principle of superposition. The “trick” to using them is almost always in coming up with correct expressions for *dl*, *dA*, or *dV*, as the case may be, expressed in terms of *r*, and also expressing the charge density function appropriately. It may be constant; it might be dependent on location.

Note carefully the meaning of *r* in these equations: It is the distance from the charge element $(q_i, \lambda dl, \sigma dA, \rho dV)$ to the location of interest, $P(x, y, z)$ (the point in space where you want to determine the field). However, don’t confuse this with the meaning of $\hat{\mathbf{r}}$; we are using it and the vector notation $\vec{\mathbf{E}}$ to write three integrals at once. That is, **Equation 5.9** is actually

$$E_x(P) = \frac{1}{4\pi\varepsilon_0}\int_{\text{line}}\left(\frac{\lambda dl}{r^2}\right)_x, \quad E_y(P) = \frac{1}{4\pi\varepsilon_0}\int_{\text{line}}\left(\frac{\lambda dl}{r^2}\right)_y, \quad E_z(P) = \frac{1}{4\pi\varepsilon_0}\int_{\text{line}}\left(\frac{\lambda dl}{r^2}\right)_z.$$

## Example 5.5

### Electric Field of a Line Segment

Find the electric field a distance *z* above the midpoint of a straight line segment of length *L* that carries a uniform line charge density $\lambda$.

### Strategy

Since this is a continuous charge distribution, we conceptually break the wire segment into differential pieces of length *dl*, each of which carries a differential amount of charge $dq = \lambda dl$. Then, we calculate the differential field created by two symmetrically placed pieces of the wire, using the symmetry of the setup to simplify the calculation (**Figure 5.23**). Finally, we integrate this differential field expression over the length of the wire (half of it, actually, as we explain below) to obtain the complete electric field expression.

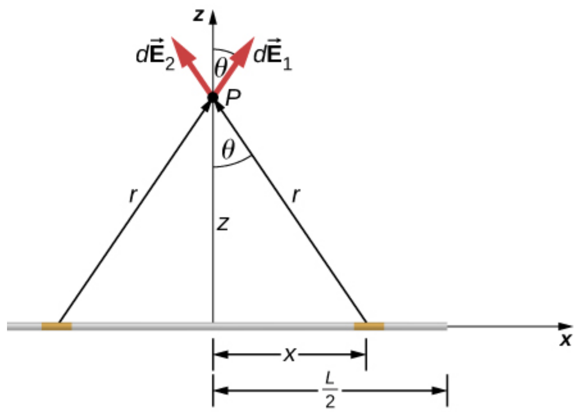



**Figure 5.23** A uniformly charged segment of wire. The electric field at point *P* can be found by applying the superposition principle to symmetrically placed charge elements and integrating.

### Solution

Before we jump into it, what do we expect the field to "look like" from far away? Since it is a finite line segment, from far away, it should look like a point charge. We will check the expression we get to see if it meets this expectation.

The electric field for a line charge is given by the general expression

$$\vec{\mathbf{E}}(P) = \frac{1}{4\pi\varepsilon_0}\int_{\text{line}}\frac{\lambda dl}{r^2}\hat{\mathbf{r}}.$$

The symmetry of the situation (our choice of the two identical differential pieces of charge) implies the horizontal (*x*)-components of the field cancel, so that the net field points in the *z*-direction. Let's check this formally.

The total field $\vec{\mathbf{E}}(P)$ is the vector sum of the fields from each of the two charge elements (call them $\vec{\mathbf{E}}_1$ and $\vec{\mathbf{E}}_2$, for now):

$$\vec{\mathbf{E}}(P) = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 = E_{1x}\hat{\mathbf{i}} + E_{1z}\hat{\mathbf{k}} + E_{2x}\left(-\hat{\mathbf{i}}\right) + E_{2z}\hat{\mathbf{k}}.$$

Because the two charge elements are identical and are the same distance away from the point $P$ where we want to calculate the field, $E_{1x} = E_{2x}$, so those components cancel. This leaves

$$\vec{\mathbf{E}}(P) = E_{1z}\hat{\mathbf{k}} + E_{2z}\hat{\mathbf{k}} = E_1 \cos\theta\hat{\mathbf{k}} + E_2 \cos\theta\hat{\mathbf{k}}.$$

These components are also equal, so we have

$$\begin{aligned}\vec{\mathbf{E}}(P) &= \frac{1}{4\pi\varepsilon_0}\int\frac{\lambda dl}{r^2}\cos\theta\hat{\mathbf{k}} + \frac{1}{4\pi\varepsilon_0}\int\frac{\lambda dl}{r^2}\cos\theta\hat{\mathbf{k}} \\ &= \frac{1}{4\pi\varepsilon_0}\int_0^{L/2}\frac{2\lambda dx}{r^2}\cos\theta\hat{\mathbf{k}}\end{aligned}$$

where our differential line element $dl$ is $dx$, in this example, since we are integrating along a line of charge that lies on the $x$-axis. (The limits of integration are 0 to $\frac{L}{2}$, not $-\frac{L}{2}$ to $+\frac{L}{2}$, because we have constructed the net field from two differential pieces of charge $dq$. If we integrated along the entire length, we would pick up an erroneous factor of 2.)

In principle, this is complete. However, to actually calculate this integral, we need to eliminate all the variables that are not given. In this case, both $r$ and $\theta$ change as we integrate outward to the end of the line charge, so those are the variables to get rid of. We can do that the same way we did for the two point charges: by noticing that

$$r = \left(z^2 + x^2\right)^{1/2}$$

and

$$\cos\theta = \frac{z}{r} = \frac{z}{\left(z^2 + x^2\right)^{1/2}}.$$

Substituting, we obtain

$$\begin{aligned}\vec{\mathbf{E}}(P) &= \frac{1}{4\pi\varepsilon_0}\int_0^{L/2}\frac{2\lambda dx}{\left(z^2 + x^2\right)}\frac{z}{\left(z^2 + x^2\right)^{1/2}}\hat{\mathbf{k}} \\ &= \frac{1}{4\pi\varepsilon_0}\int_0^{L/2}\frac{2\lambda z}{\left(z^2 + x^2\right)^{3/2}}dx\hat{\mathbf{k}} \\ &= \frac{2\lambda z}{4\pi\varepsilon_0}\left[\frac{x}{z^2\sqrt{z^2 + x^2}}\right]\Bigg|_0^{L/2}\hat{\mathbf{k}}\end{aligned}$$

which simplifies to

$$\vec{\mathbf{E}}(z) = \frac{1}{4\pi\varepsilon_0}\frac{\lambda L}{z\sqrt{z^2 + \frac{L^2}{4}}}\hat{\mathbf{k}}. \tag{5.12}$$

### Significance

Notice, once again, the use of symmetry to simplify the problem. This is a very common strategy for calculating electric fields. The fields of nonsymmetrical charge distributions have to be handled with multiple integrals and may need to be calculated numerically by a computer.

**5.4 Check Your Understanding** How would the strategy used above change to calculate the electric field at a point a distance $z$ above one end of the finite line segment?

## Example 5.6

### Electric Field of an Infinite Line of Charge

Find the electric field a distance $z$ above the midpoint of an infinite line of charge that carries a uniform line charge density $\lambda$.

### Strategy

This is exactly like the preceding example, except the limits of integration will be $-\infty$ to $+\infty$.

### Solution

Again, the horizontal components cancel out, so we wind up with

$$\vec{\mathbf{E}}(P) = \frac{1}{4\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{\lambda dx}{r^2}\cos\theta\,\hat{\mathbf{k}}$$

where our differential line element $dl$ is $dx$, in this example, since we are integrating along a line of charge that lies on the $x$-axis. Again,

$$\cos\theta = \frac{z}{r} = \frac{z}{\left(z^2+x^2\right)^{1/2}}.$$

Substituting, we obtain

$$\begin{aligned}\vec{\mathbf{E}}(P) &= \frac{1}{4\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{\lambda dx}{\left(z^2+x^2\right)}\frac{z}{\left(z^2+x^2\right)^{1/2}}\hat{\mathbf{k}} \\ &= \frac{1}{4\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{\lambda z}{\left(z^2+x^2\right)^{3/2}}dx\,\hat{\mathbf{k}} \\ &= \frac{\lambda z}{4\pi\varepsilon_0}\left[\frac{x}{z^2\sqrt{z^2+x^2}}\right]_{-\infty}^{\infty}\hat{\mathbf{k}},\end{aligned}$$

which simplifies to

$$\vec{\mathbf{E}}(z) = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{z}\hat{\mathbf{k}}.$$

### Significance

Our strategy for working with continuous charge distributions also gives useful results for charges with infinite dimension.

In the case of a finite line of charge, note that for $z \gg L$, $z^2$ dominates the $L$ in the denominator, so that **Equation 5.12** simplifies to

$$\vec{\mathbf{E}} \approx \frac{1}{4\pi\varepsilon_0}\frac{\lambda L}{z^2}\hat{\mathbf{k}}.$$

If you recall that $\lambda L = q$, the total charge on the wire, we have retrieved the expression for the field of a point charge, as expected.

In the limit $L \to \infty$, on the other hand, we get the field of an **infinite straight wire**, which is a straight wire whose length is much, much greater than either of its other dimensions, and also much, much greater than the distance at which the field is to be calculated:

$$\vec{\mathbf{E}}(z) = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{z}\hat{\mathbf{k}}. \tag{5.13}$$

An interesting artifact of this infinite limit is that we have lost the usual $1/r^2$ dependence that we are used to. This will become even more intriguing in the case of an infinite plane.

## Example 5.7

### Electric Field due to a Ring of Charge

A ring has a uniform charge density $\lambda$, with units of coulomb per unit meter of arc. Find the electric potential at a point on the axis passing through the center of the ring.

### Strategy

We use the same procedure as for the charged wire. The difference here is that the charge is distributed on a circle. We divide the circle into infinitesimal elements shaped as arcs on the circle and use polar coordinates shown in **Figure 5.24**.

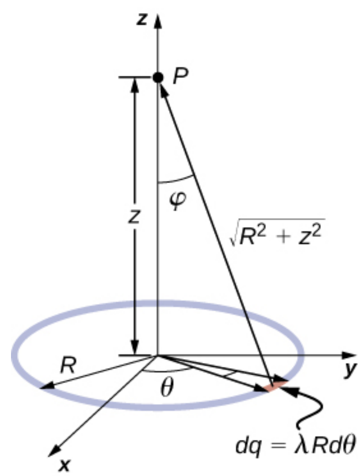



**Figure 5.24** The system and variable for calculating the electric field due to a ring of charge.

### Solution

The electric field for a line charge is given by the general expression

$$\vec{\mathbf{E}}(P) = \frac{1}{4\pi\varepsilon_0}\int_{\text{line}}\frac{\lambda dl}{r^2}\hat{\mathbf{r}}.$$

A general element of the arc between $\theta$ and $\theta + d\theta$ is of length $Rd\theta$ and therefore contains a charge equal to $\lambda R d\theta$. The element is at a distance of $r = \sqrt{z^2 + R^2}$ from $P$, the angle is $\cos\phi = \frac{z}{\sqrt{z^2 + R^2}}$, and therefore the electric field is

$$\begin{aligned}\vec{\mathbf{E}}(P) &= \frac{1}{4\pi\varepsilon_0}\int_{\text{line}} \frac{\lambda dl}{r^2}\hat{\mathbf{r}} = \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi} \frac{\lambda R d\theta}{z^2 + R^2}\frac{z}{\sqrt{z^2 + R^2}}\hat{\mathbf{z}} \\ &= \frac{1}{4\pi\varepsilon_0}\frac{\lambda R z}{\left(z^2 + R^2\right)^{3/2}}\hat{\mathbf{z}}\int_0^{2\pi} d\theta = \frac{1}{4\pi\varepsilon_0}\frac{2\pi\lambda R z}{\left(z^2 + R^2\right)^{3/2}}\hat{\mathbf{z}} \\ &= \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{tot}} z}{\left(z^2 + R^2\right)^{3/2}}\hat{\mathbf{z}}.\end{aligned}$$

### Significance

As usual, symmetry simplified this problem, in this particular case resulting in a trivial integral. Also, when we take the limit of $z >> R$, we find that

$$\vec{\mathbf{E}} \approx \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{tot}}}{z^2}\hat{\mathbf{z}},$$

as we expect.

## Example 5.8

### The Field of a Disk

Find the electric field of a circular thin disk of radius *R* and uniform charge density at a distance *z* above the center of the disk (**Figure 5.25**)

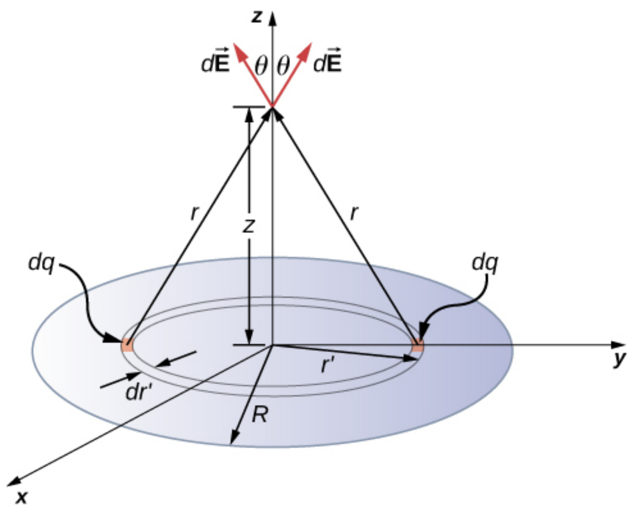



**Figure 5.25** A uniformly charged disk. As in the line charge example, the field above the center of this disk can be calculated by taking advantage of the symmetry of the charge distribution.

### Strategy

The electric field for a surface charge is given by

$$\vec{\mathbf{E}}(P) = \frac{1}{4\pi\varepsilon_0}\int_{\text{surface}} \frac{\sigma dA}{r^2}\hat{\mathbf{r}}.$$

To solve surface charge problems, we break the surface into symmetrical differential "stripes" that match the shape of the surface; here, we'll use rings, as shown in the figure. Again, by symmetry, the horizontal components cancel and the field is entirely in the vertical $(\hat{\mathbf{k}})$ direction. The vertical component of the electric field is extracted by multiplying by $\cos\theta$, so

$$\vec{\mathbf{E}}(P) = \frac{1}{4\pi\varepsilon_0}\int_{\text{surface}} \frac{\sigma dA}{r^2}\cos\theta\,\hat{\mathbf{k}}.$$

As before, we need to rewrite the unknown factors in the integrand in terms of the given quantities. In this case,

$$\begin{aligned} dA &= 2\pi r' dr' \\ r^2 &= r'^2 + z^2 \\ \cos\theta &= \frac{z}{\left(r'^2+z^2\right)^{1/2}}. \end{aligned}$$

(Please take note of the two different "$r$'s" here; $r$ is the distance from the differential ring of charge to the point $P$ where we wish to determine the field, whereas $r'$ is the distance from the center of the disk to the differential ring of charge.) Also, we already performed the polar angle integral in writing down $dA$.

### Solution

Substituting all this in, we get

$$\begin{aligned} \vec{\mathbf{E}}(P) &= \vec{\mathbf{E}}(z) = \frac{1}{4\pi\varepsilon_0}\int_0^R \frac{\sigma(2\pi r' dr')z}{\left(r'^2+z^2\right)^{3/2}}\hat{\mathbf{k}} \\ &= \frac{1}{4\pi\varepsilon_0}(2\pi\sigma z)\left(\frac{1}{z} - \frac{1}{\sqrt{R^2+z^2}}\right)\hat{\mathbf{k}} \end{aligned}$$

or, more simply,

$$\vec{\mathbf{E}}(z) = \frac{1}{4\pi\varepsilon_0}\left(2\pi\sigma - \frac{2\pi\sigma z}{\sqrt{R^2+z^2}}\right)\hat{\mathbf{k}}. \tag{5.14}$$

### Significance

Again, it can be shown (via a Taylor expansion) that when $z \gg R$, this reduces to

$$\vec{\mathbf{E}}(z) \approx \frac{1}{4\pi\varepsilon_0}\frac{\sigma\pi R^2}{z^2}\hat{\mathbf{k}},$$

which is the expression for a point charge $Q = \sigma\pi R^2$.

**5.5 Check Your Understanding** How would the above limit change with a uniformly charged rectangle instead of a disk?

As $R \to \infty$, **Equation 5.14** reduces to the field of an **infinite plane**, which is a flat sheet whose area is much, much greater than its thickness, and also much, much greater than the distance at which the field is to be calculated:

$$\vec{\mathbf{E}} = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{k}}. \tag{5.15}$$

Note that this field is constant. This surprising result is, again, an artifact of our limit, although one that we will make use of repeatedly in the future. To understand why this happens, imagine being placed above an infinite plane of constant charge.

Does the plane look any different if you vary your altitude? No—you still see the plane going off to infinity, no matter how far you are from it. It is important to note that **Equation 5.15** is because we are above the plane. If we were below, the field would point in the $-\hat{\mathbf{k}}$ direction.

## Example 5.9

### The Field of Two Infinite Planes

Find the electric field everywhere resulting from two infinite planes with equal but opposite charge densities (**Figure 5.26**).

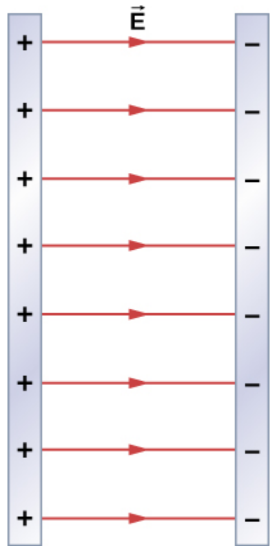



**Figure 5.26** Two charged infinite planes. Note the direction of the electric field.

### Strategy

We already know the electric field resulting from a single infinite plane, so we may use the principle of superposition to find the field from two.

### Solution

The electric field points away from the positively charged plane and toward the negatively charged plane. Since the $\sigma$ are equal and opposite, this means that in the region outside of the two planes, the electric fields cancel each other out to zero.

However, in the region between the planes, the electric fields add, and we get

$$\vec{\mathbf{E}} = \frac{\sigma}{\varepsilon_0}\hat{\mathbf{i}}$$

for the electric field. The $\hat{\mathbf{i}}$ is because in the figure, the field is pointing in the $+x$-direction.

### Significance

Systems that may be approximated as two infinite planes of this sort provide a useful means of creating uniform electric fields.

**5.6 Check Your Understanding** What would the electric field look like in a system with two parallel positively charged planes with equal charge densities?

# 5.6 | Electric Field Lines

## Learning Objectives

By the end of this section, you will be able to:

- Explain the purpose of an electric field diagram
- Describe the relationship between a vector diagram and a field line diagram
- Explain the rules for creating a field diagram and why these rules make physical sense
- Sketch the field of an arbitrary source charge

Now that we have some experience calculating electric fields, let's try to gain some insight into the geometry of electric fields. As mentioned earlier, our model is that the charge on an object (the source charge) alters space in the region around it in such a way that when another charged object (the test charge) is placed in that region of space, that test charge experiences an electric force. The concept of electric **field lines**, and of electric field line diagrams, enables us to visualize the way in which the space is altered, allowing us to visualize the field. The purpose of this section is to enable you to create sketches of this geometry, so we will list the specific steps and rules involved in creating an accurate and useful sketch of an electric field.

It is important to remember that electric fields are three-dimensional. Although in this book we include some pseudo-three-dimensional images, several of the diagrams that you'll see (both here, and in subsequent chapters) will be two-dimensional projections, or cross-sections. Always keep in mind that in fact, you're looking at a three-dimensional phenomenon.

Our starting point is the physical fact that the electric field of the source charge causes a test charge in that field to experience a force. By definition, electric field vectors point in the same direction as the electric force that a (hypothetical) positive test charge would experience, if placed in the field (Figure 5.27)

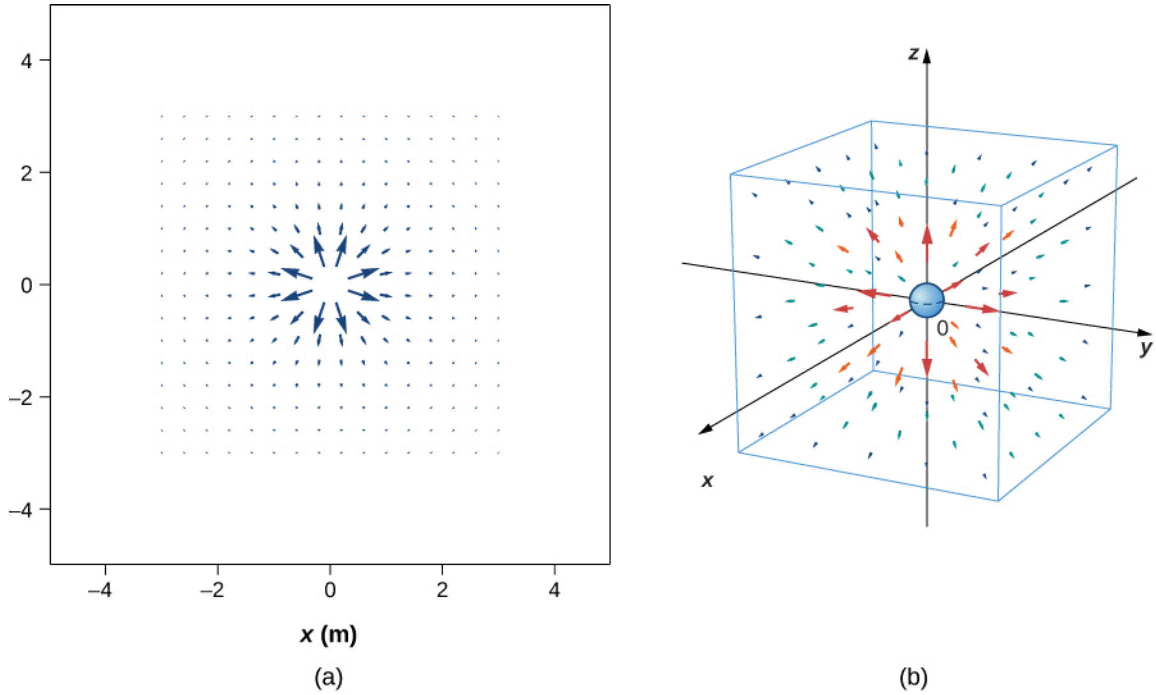



**Figure 5.27** The electric field of a positive point charge. A large number of field vectors are shown. Like all vector arrows, the length of each vector is proportional to the magnitude of the field at each point. (a) Field in two dimensions; (b) field in three dimensions.

We've plotted many field vectors in the figure, which are distributed uniformly around the source charge. Since the electric field is a vector, the arrows that we draw correspond at every point in space to both the magnitude and the direction of the field at that point. As always, the length of the arrow that we draw corresponds to the magnitude of the field vector at that point. For a point source charge, the length decreases by the square of the distance from the source charge. In addition, the direction of the field vector is radially away from the source charge, because the direction of the electric field is defined by the direction of the force that a positive test charge would experience in that field. (Again, keep in mind that the actual field is three-dimensional; there are also field lines pointing out of and into the page.)

This diagram is correct, but it becomes less useful as the source charge distribution becomes more complicated. For example, consider the vector field diagram of a dipole (**Figure 5.28**).

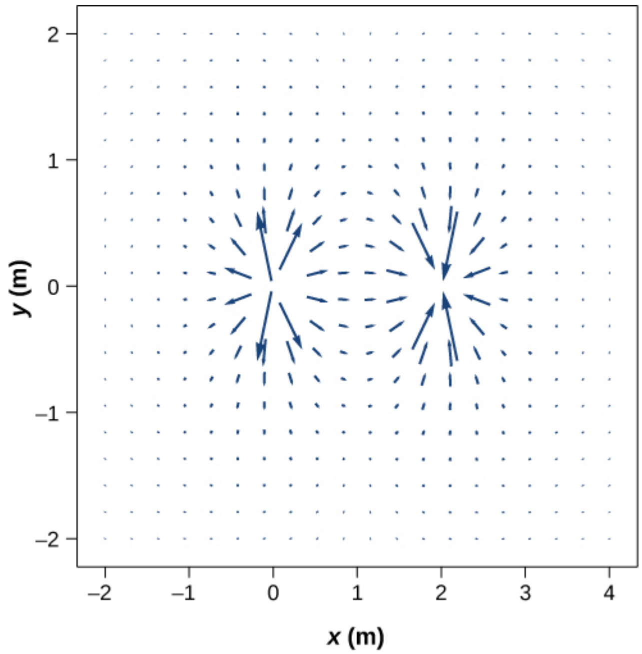



**Figure 5.28** The vector field of a dipole. Even with just two identical charges, the vector field diagram becomes difficult to understand.

There is a more useful way to present the same information. Rather than drawing a large number of increasingly smaller vector arrows, we instead connect all of them together, forming continuous lines and curves, as shown in **Figure 5.29**.

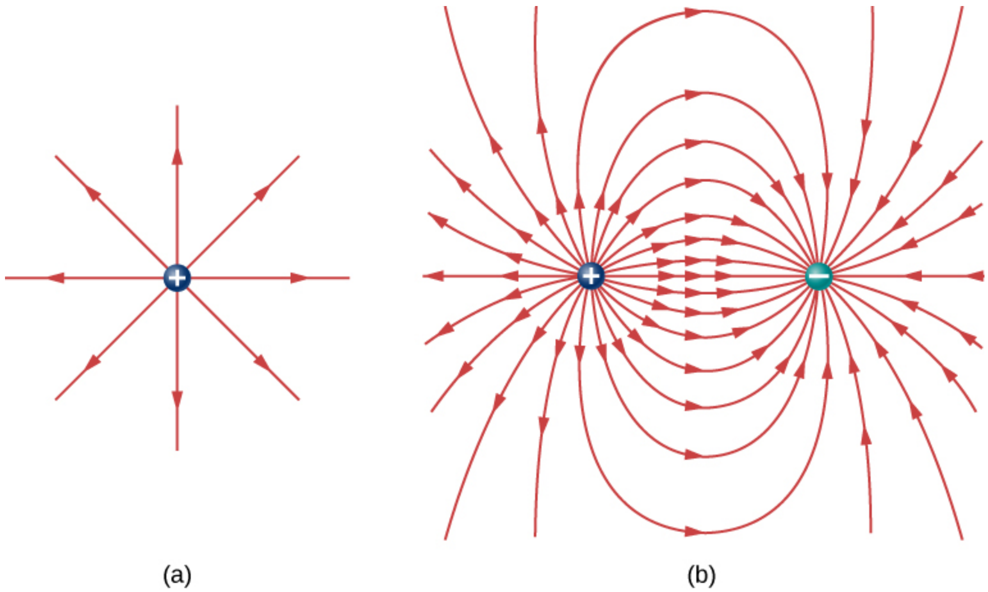



**Figure 5.29** (a) The electric field line diagram of a positive point charge. (b) The field line diagram of a dipole. In both diagrams, the magnitude of the field is indicated by the field line density. The field *vectors* (not shown here) are everywhere tangent to the field lines.

Although it may not be obvious at first glance, these field diagrams convey the same information about the electric field as do the vector diagrams. First, the direction of the field at every point is simply the direction of the field vector at that same point. In other words, at any point in space, the field vector at each point is tangent to the field line at that same point. The arrowhead placed on a field line indicates its direction.

As for the magnitude of the field, that is indicated by the **field line density**—that is, the number of field lines per unit area passing through a small cross-sectional area perpendicular to the electric field. This field line density is drawn to be proportional to the magnitude of the field at that cross-section. As a result, if the field lines are close together (that is, the field line density is greater), this indicates that the magnitude of the field is large at that point. If the field lines are far apart at the cross-section, this indicates the magnitude of the field is small. **Figure 5.30** shows the idea.

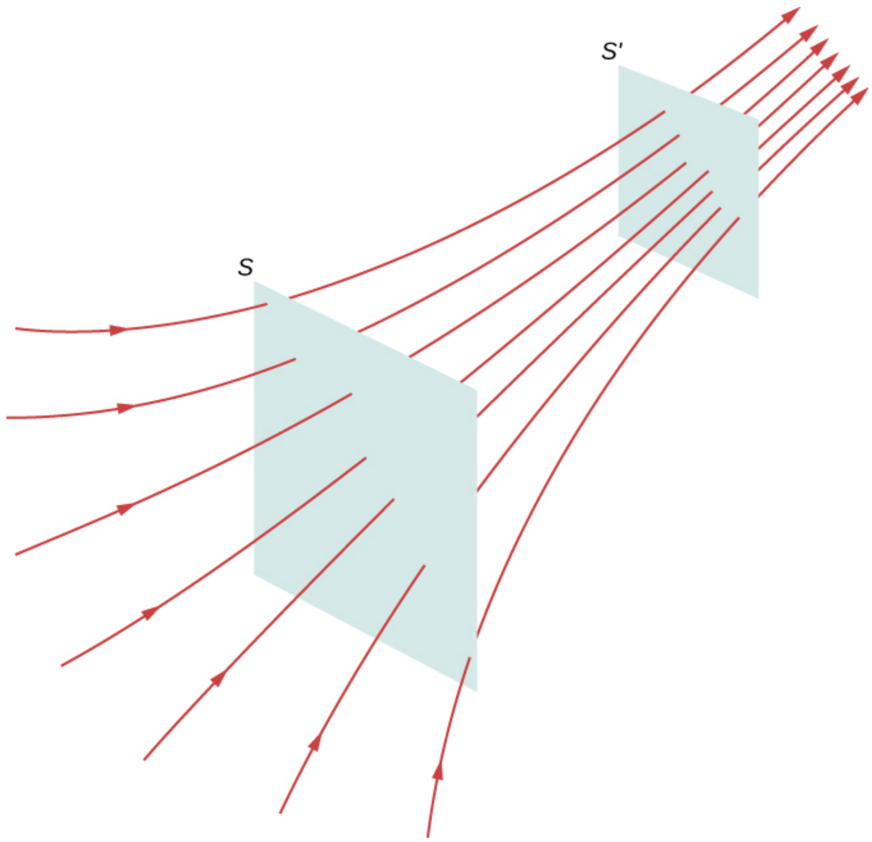



**Figure 5.30** Electric field lines passing through imaginary areas. Since the number of lines passing through each area is the same, but the areas themselves are different, the field line density is different. This indicates different magnitudes of the electric field at these points.

In **Figure 5.30**, the same number of field lines passes through both surfaces ($S$ and $S'$), but the surface $S$ is larger than surface $S'$. Therefore, the density of field lines (number of lines per unit area) is larger at the location of $S'$, indicating that the electric field is stronger at the location of $S'$ than at $S$. The rules for creating an electric field diagram are as follows.

## Problem-Solving Strategy: Drawing Electric Field Lines

1. Electric field lines either originate on positive charges or come in from infinity, and either terminate on negative charges or extend out to infinity.
2. The number of field lines originating or terminating at a charge is proportional to the magnitude of that charge. A charge of $2q$ will have twice as many lines as a charge of $q$.
3. At every point in space, the field vector at that point is tangent to the field line at that same point.

4. The field line density at any point in space is proportional to (and therefore is representative of) the magnitude of the field at that point in space.
5. Field lines can never cross. Since a field line represents the direction of the field at a given point, if two field lines crossed at some point, that would imply that the electric field was pointing in two different directions at a single point. This in turn would suggest that the (net) force on a test charge placed at that point would point in two different directions. Since this is obviously impossible, it follows that field lines must never cross.

Always keep in mind that field lines serve only as a convenient way to visualize the electric field; they are not physical entities. Although the direction and relative intensity of the electric field can be deduced from a set of field lines, the lines can also be misleading. For example, the field lines drawn to represent the electric field in a region must, by necessity, be discrete. However, the actual electric field in that region exists at every point in space.

Field lines for three groups of discrete charges are shown in **Figure 5.31**. Since the charges in parts (a) and (b) have the same magnitude, the same number of field lines are shown starting from or terminating on each charge. In (c), however, we draw three times as many field lines leaving the $+3q$ charge as entering the $-q$. The field lines that do not terminate at $-q$ emanate outward from the charge configuration, to infinity.

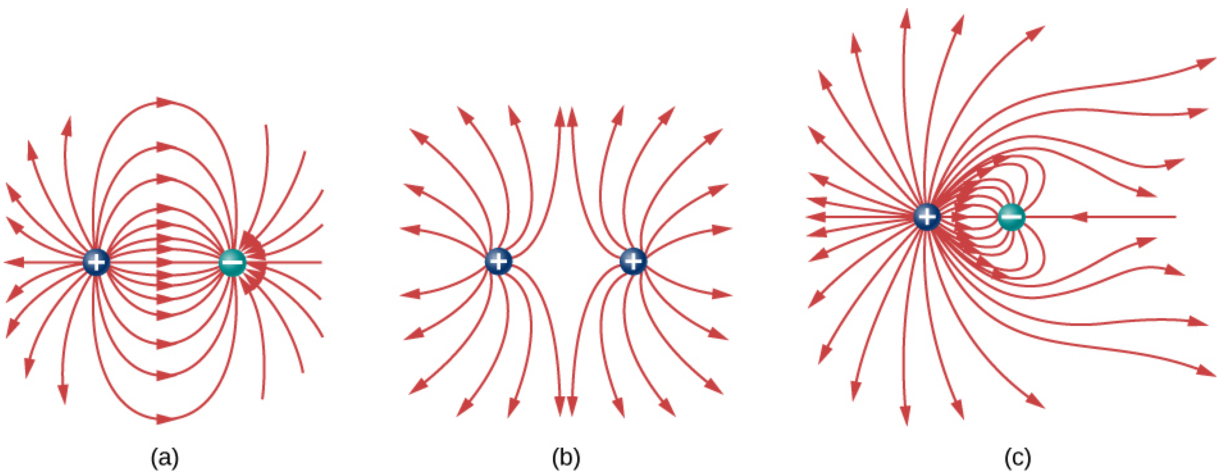



**Figure 5.31** Three typical electric field diagrams. (a) A dipole. (b) Two identical charges. (c) Two charges with opposite signs and different magnitudes. Can you tell from the diagram which charge has the larger magnitude?

The ability to construct an accurate electric field diagram is an important, useful skill; it makes it much easier to estimate, predict, and therefore calculate the electric field of a source charge. The best way to develop this skill is with software that allows you to place source charges and then will draw the net field upon request. We strongly urge you to search the Internet for a program. Once you've found one you like, run several simulations to get the essential ideas of field diagram construction. Then practice drawing field diagrams, and checking your predictions with the computer-drawn diagrams.

One example of a **field-line drawing program (https://openstaxcollege.org/l/21fieldlindrapr)** is from the PhET "Charges and Fields" simulation.

# 5.7 | Electric Dipoles

## Learning Objectives

By the end of this section, you will be able to:

- Describe a permanent dipole
- Describe an induced dipole
- Define and calculate an electric dipole moment
- Explain the physical meaning of the dipole moment

Earlier we discussed, and calculated, the electric field of a dipole: two equal and opposite charges that are "close" to each other. (In this context, "close" means that the distance $d$ between the two charges is much, much less than the distance of the field point $P$, the location where you are calculating the field.) Let's now consider what happens to a dipole when it is placed in an external field $\vec{\mathbf{E}}$. We assume that the dipole is a **permanent dipole**; it exists without the field, and does not break apart in the external field.

## Rotation of a Dipole due to an Electric Field

For now, we deal with only the simplest case: The external field is uniform in space. Suppose we have the situation depicted in **Figure 5.32**, where we denote the distance between the charges as the vector $\vec{\mathbf{d}}$, pointing from the negative charge to the positive charge. The forces on the two charges are equal and opposite, so there is no net force on the dipole. However, there is a torque:

$$\begin{aligned}\vec{\tau} &= \left(\frac{\vec{\mathbf{d}}}{2} \times \vec{\mathbf{F}}_+\right) + \left(-\frac{\vec{\mathbf{d}}}{2} \times \vec{\mathbf{F}}_-\right) \\ &= \left[\left(\frac{\vec{\mathbf{d}}}{2}\right) \times \left(+q\vec{\mathbf{E}}\right) + \left(-\frac{\vec{\mathbf{d}}}{2}\right) \times \left(-q\vec{\mathbf{E}}\right)\right] \\ &= q\vec{\mathbf{d}} \times \vec{\mathbf{E}}.\end{aligned}$$

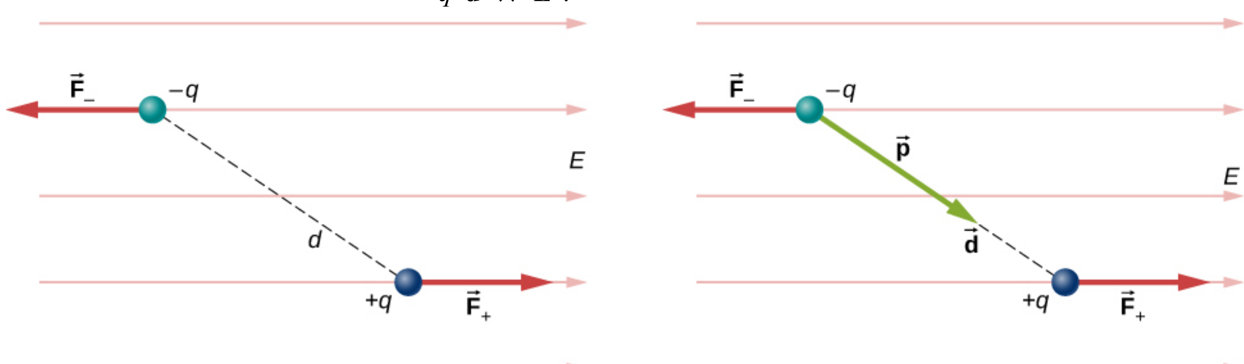



**Figure 5.32** A dipole in an external electric field. (a) The net force on the dipole is zero, but the net torque is not. As a result, the dipole rotates, becoming aligned with the external field. (b) The dipole moment is a convenient way to characterize this effect. The $\vec{\mathbf{d}}$ points in the same direction as $\vec{\mathbf{p}}$.

The quantity $q\vec{\mathbf{d}}$ (the magnitude of each charge multiplied by the vector distance between them) is a property of the dipole; its value, as you can see, determines the torque that the dipole experiences in the external field. It is useful, therefore, to define this product as the so-called **dipole moment** of the dipole:

$$\vec{\mathbf{p}} \equiv q\vec{\mathbf{d}}. \tag{5.16}$$

We can therefore write

$$\vec{\tau} = \vec{\mathbf{p}} \times \vec{\mathbf{E}} . \tag{5.17}$$

Recall that a torque changes the angular velocity of an object, the dipole, in this case. In this situation, the effect is to rotate the dipole (that is, align the direction of $\vec{\mathbf{p}}$ ) so that it is parallel to the direction of the external field.

## Induced Dipoles

Neutral atoms are, by definition, electrically neutral; they have equal amounts of positive and negative charge. Furthermore, since they are spherically symmetrical, they do not have a "built-in" dipole moment the way most asymmetrical molecules do. They obtain one, however, when placed in an external electric field, because the external field causes oppositely directed forces on the positive nucleus of the atom versus the negative electrons that surround the nucleus. The result is a new charge distribution of the atom, and therefore, an **induced dipole** moment (Figure 5.33).

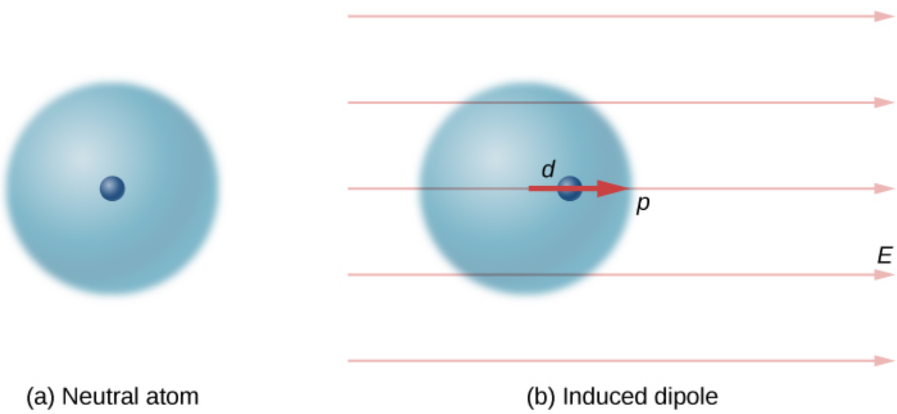



Figure 5.33 A dipole is induced in a neutral atom by an external electric field. The induced dipole moment is aligned with the external field.

An important fact here is that, just as for a rotated polar molecule, the result is that the dipole moment ends up aligned parallel to the external electric field. Generally, the magnitude of an induced dipole is much smaller than that of an inherent dipole. For both kinds of dipoles, notice that once the alignment of the dipole (rotated or induced) is complete, the net effect is to decrease the total electric field $\vec{\mathbf{E}}_{\text{total}} = \vec{\mathbf{E}}_{\text{external}} + \vec{\mathbf{E}}_{\text{dipole}}$ in the regions inside the dipole charges (Figure 5.34). By "inside" we mean in between the charges. This effect is crucial for capacitors, as you will see in Capacitance.

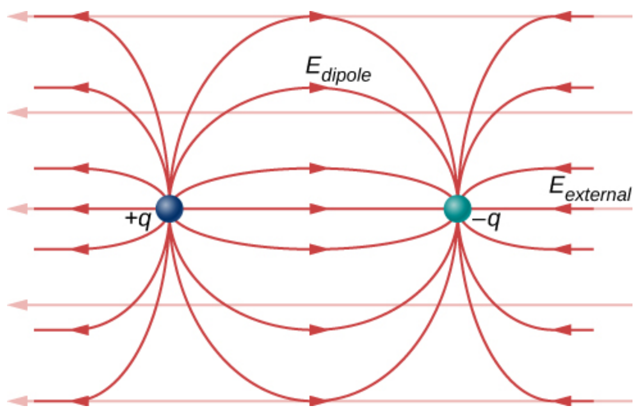



Figure 5.34 The net electric field is the vector sum of the field of the dipole plus the external field.

Recall that we found the electric field of a dipole in Equation 5.7. If we rewrite it in terms of the dipole moment we get:

$$\vec{\mathbf{E}}(z) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{\mathbf{p}}}{z^3}.$$

The form of this field is shown in **Figure 5.34**. Notice that along the plane perpendicular to the axis of the dipole and midway between the charges, the direction of the electric field is opposite that of the dipole and gets weaker the further from the axis one goes. Similarly, on the axis of the dipole (but outside it), the field points in the same direction as the dipole, again getting weaker the further one gets from the charges.

# CHAPTER 5 REVIEW

## KEY TERMS

**charging by induction** process by which an electrically charged object brought near a neutral object creates a charge separation in that object

**conduction electron** electron that is free to move away from its atomic orbit

**conductor** material that allows electrons to move separately from their atomic orbits; object with properties that allow charges to move about freely within it

**continuous charge distribution** total source charge composed of so large a number of elementary charges that it must be treated as continuous, rather than discrete

**coulomb** SI unit of electric charge

**Coulomb force** another term for the electrostatic force

**Coulomb's law** mathematical equation calculating the electrostatic force vector between two charged particles

**dipole** two equal and opposite charges that are fixed close to each other

**dipole moment** property of a dipole; it characterizes the combination of distance between the opposite charges, and the magnitude of the charges

**electric charge** physical property of an object that causes it to be attracted toward or repelled from another charged object; each charged object generates and is influenced by a force called an electric force

**electric field** physical phenomenon created by a charge; it "transmits" a force between a two charges

**electric force** noncontact force observed between electrically charged objects

**electron** particle surrounding the nucleus of an atom and carrying the smallest unit of negative charge

**electrostatic attraction** phenomenon of two objects with opposite charges attracting each other

**electrostatic force** amount and direction of attraction or repulsion between two charged bodies; the assumption is that the source charges remain motionless

**electrostatic repulsion** phenomenon of two objects with like charges repelling each other

**electrostatics** study of charged objects which are not in motion

**field line** smooth, usually curved line that indicates the direction of the electric field

**field line density** number of field lines per square meter passing through an imaginary area; its purpose is to indicate the field strength at different points in space

**induced dipole** typically an atom, or a spherically symmetric molecule; a dipole created due to opposite forces displacing the positive and negative charges

**infinite plane** flat sheet in which the dimensions making up the area are much, much greater than its thickness, and also much, much greater than the distance at which the field is to be calculated; its field is constant

**infinite straight wire** straight wire whose length is much, much greater than either of its other dimensions, and also much, much greater than the distance at which the field is to be calculated

**insulator** material that holds electrons securely within their atomic orbits

**ion** atom or molecule with more or fewer electrons than protons

**law of conservation of charge** net electric charge of a closed system is constant

**linear charge density** amount of charge in an element of a charge distribution that is essentially one-dimensional (the width and height are much, much smaller than its length); its units are C/m

**neutron** neutral particle in the nucleus of an atom, with (nearly) the same mass as a proton

**permanent dipole** typically a molecule; a dipole created by the arrangement of the charged particles from which the dipole is created

**permittivity of vacuum** also called the permittivity of free space, and constant describing the strength of the electric force in a vacuum

**polarization** slight shifting of positive and negative charges to opposite sides of an object

**principle of superposition** useful fact that we can simply add up all of the forces due to charges acting on an object

**proton** particle in the nucleus of an atom and carrying a positive charge equal in magnitude to the amount of negative charge carried by an electron

**static electricity** buildup of electric charge on the surface of an object; the arrangement of the charge remains constant (“static”)

**superposition** concept that states that the net electric field of multiple source charges is the vector sum of the field of each source charge calculated individually

**surface charge density** amount of charge in an element of a two-dimensional charge distribution (the thickness is small); its units are $\mathrm{C/m^2}$

**volume charge density** amount of charge in an element of a three-dimensional charge distribution; its units are $\mathrm{C/m^3}$

# KEY EQUATIONS

| | |
|---|---|
| Coulomb’s law | $\vec{\mathbf{F}}_{12}(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12}$ |
| Superposition of electric forces | $\vec{\mathbf{F}}(r) = \frac{1}{4\pi\varepsilon_0} Q \sum_{i=1}^{N}\frac{q_i}{r_i^2}\hat{\mathbf{r}}_i$ |
| Electric force due to an electric field | $\vec{\mathbf{F}} = Q\vec{\mathbf{E}}$ |
| Electric field at point *P* | $\vec{\mathbf{E}}(P) \equiv \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_i^2}\hat{\mathbf{r}}_i$ |
| Field of an infinite wire | $\vec{\mathbf{E}}(z) = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{z}\hat{\mathbf{k}}$ |
| Field of an infinite plane | $\vec{\mathbf{E}} = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{k}}$ |
| Dipole moment | $\vec{\mathbf{p}} \equiv q\vec{\mathbf{d}}$ |
| Torque on dipole in external E-field | $\vec{\tau} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$ |

# SUMMARY

## 5.1 Electric Charge

- There are only two types of charge, which we call positive and negative. Like charges repel, unlike charges attract, and the force between charges decreases with the square of the distance.
- The vast majority of positive charge in nature is carried by protons, whereas the vast majority of negative charge is carried by electrons. The electric charge of one electron is equal in magnitude and opposite in sign to the charge of one proton.
- An ion is an atom or molecule that has nonzero total charge due to having unequal numbers of electrons and protons.
- The SI unit for charge is the coulomb (C), with protons and electrons having charges of opposite sign but equal magnitude; the magnitude of this basic charge is $e \equiv 1.602 \times 10^{-19}\ \mathrm{C}$

- Both positive and negative charges exist in neutral objects and can be separated by bringing the two objects into physical contact; rubbing the objects together can remove electrons from the bonds in one object and place them on the other object, increasing the charge separation.
- For macroscopic objects, negatively charged means an excess of electrons and positively charged means a depletion of electrons.
- The law of conservation of charge states that the net charge of a closed system is constant.

### 5.2 Conductors, Insulators, and Charging by Induction

- A conductor is a substance that allows charge to flow freely through its atomic structure.
- An insulator holds charge fixed in place.
- Polarization is the separation of positive and negative charges in a neutral object. Polarized objects have their positive and negative charges concentrated in different areas, giving them a charge distribution.

### 5.3 Coulomb's Law

- Coulomb’s law gives the magnitude of the force between point charges. It is

$$\vec{\mathbf{F}}_{12}(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12}$$

  where $q_2$ and $q_2$ are two point charges separated by a distance $r$. This Coulomb force is extremely basic, since most charges are due to point-like particles. It is responsible for all electrostatic effects and underlies most macroscopic forces.

### 5.4 Electric Field

- The electric field is an alteration of space caused by the presence of an electric charge. The electric field mediates the electric force between a source charge and a test charge.
- The electric field, like the electric force, obeys the superposition principle
- The field is a vector; by definition, it points away from positive charges and toward negative charges.

### 5.5 Calculating Electric Fields of Charge Distributions

- A very large number of charges can be treated as a continuous charge distribution, where the calculation of the field requires integration. Common cases are:
  - one-dimensional (like a wire); uses a line charge density $\lambda$
  - two-dimensional (metal plate); uses surface charge density $\sigma$
  - three-dimensional (metal sphere); uses volume charge density $\rho$
- The “source charge” is a differential amount of charge $dq$. Calculating $dq$ depends on the type of source charge distribution:

$$dq = \lambda dl; \quad dq = \sigma dA; \quad dq = \rho dV.$$

- Symmetry of the charge distribution is usually key.
- Important special cases are the field of an “infinite” wire and the field of an “infinite” plane.

### 5.6 Electric Field Lines

- Electric field diagrams assist in visualizing the field of a source charge.
- The magnitude of the field is proportional to the field line density.
- Field vectors are everywhere tangent to field lines.

### 5.7 Electric Dipoles

- If a permanent dipole is placed in an external electric field, it results in a torque that aligns it with the external field.
- If a nonpolar atom (or molecule) is placed in an external field, it gains an induced dipole that is aligned with the external field.
- The net field is the vector sum of the external field plus the field of the dipole (physical or induced).
- The strength of the polarization is described by the dipole moment of the dipole, $\vec{\mathbf{p}} = q\vec{\mathbf{d}}$ .

## CONCEPTUAL QUESTIONS

### 5.1 Electric Charge

**1.** There are very large numbers of charged particles in most objects. Why, then, don't most objects exhibit static electricity?

**2.** Why do most objects tend to contain nearly equal numbers of positive and negative charges?

**3.** A positively charged rod attracts a small piece of cork. (a) Can we conclude that the cork is negatively charged? (b) The rod repels another small piece of cork. Can we conclude that this piece is positively charged?

**4.** Two bodies attract each other electrically. Do they both have to be charged? Answer the same question if the bodies repel one another.

**5.** How would you determine whether the charge on a particular rod is positive or negative?

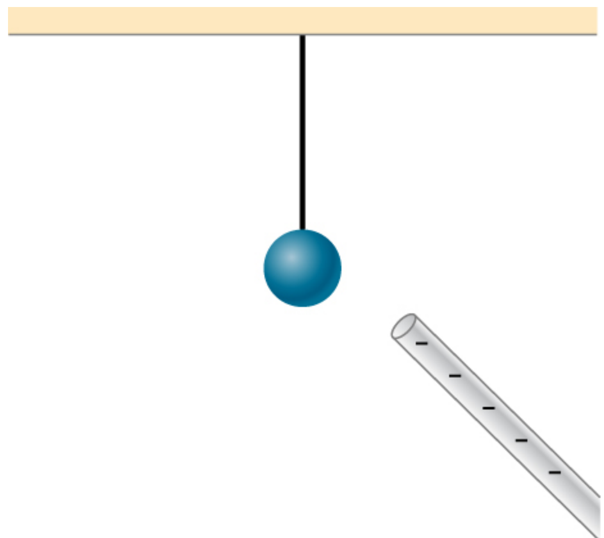
### 5.2 Conductors, Insulators, and Charging by Induction

**6.** An eccentric inventor attempts to levitate a cork ball by wrapping it with foil and placing a large negative charge on the ball and then putting a large positive charge on the ceiling of his workshop. Instead, while attempting to place a large negative charge on the ball, the foil flies off. Explain.

**7.** When a glass rod is rubbed with silk, it becomes positive and the silk becomes negative—yet both attract dust. Does the dust have a third type of charge that is attracted to both positive and negative? Explain.

**8.** Why does a car always attract dust right after it is polished? (Note that car wax and car tires are insulators.)

**9.** Does the uncharged conductor shown below experience a net electric force?




**10.** While walking on a rug, a person frequently becomes charged because of the rubbing between his shoes and the rug. This charge then causes a spark and a slight shock when the person gets close to a metal object. Why are these shocks so much more common on a dry day?

**11.** Compare charging by conduction to charging by induction.

**12.** Small pieces of tissue are attracted to a charged comb. Soon after sticking to the comb, the pieces of tissue are repelled from it. Explain.

**13.** Trucks that carry gasoline often have chains dangling from their undercarriages and brushing the ground. Why?

**14.** Why do electrostatic experiments work so poorly in humid weather?

**15.** Why do some clothes cling together after being removed from the clothes dryer? Does this happen if they're still damp?

**16.** Can induction be used to produce charge on an insulator?

**17.** Suppose someone tells you that rubbing quartz with cotton cloth produces a third kind of charge on the quartz. Describe what you might do to test this claim.

**18.** A handheld copper rod does not acquire a charge when you rub it with a cloth. Explain why.

**19.** Suppose you place a charge $q$ near a large metal plate. (a) If $q$ is attracted to the plate, is the plate necessarily charged? (b) If $q$ is repelled by the plate, is the plate necessarily charged?

### 5.3 Coulomb's Law

**20.** Would defining the charge on an electron to be positive have any effect on Coulomb's law?

**21.** An atomic nucleus contains positively charged protons and uncharged neutrons. Since nuclei do stay together, what must we conclude about the forces between these nuclear particles?

**22.** Is the force between two fixed charges influenced by the presence of other charges?

### 5.4 Electric Field

**23.** When measuring an electric field, could we use a negative rather than a positive test charge?

**24.** During fair weather, the electric field due to the net charge on Earth points downward. Is Earth charged positively or negatively?

**25.** If the electric field at a point on the line between two charges is zero, what do you know about the charges?

**26.** Two charges lie along the $x$-axis. Is it true that the net electric field always vanishes at some point (other than infinity) along the $x$-axis?

### 5.5 Calculating Electric Fields of Charge Distributions

**27.** Give a plausible argument as to why the electric field outside an infinite charged sheet is constant.

**28.** Compare the electric fields of an infinite sheet of charge, an infinite, charged conducting plate, and infinite, oppositely charged parallel plates.

**29.** Describe the electric fields of an infinite charged plate and of two infinite, charged parallel plates in terms of the electric field of an infinite sheet of charge.

**30.** A negative charge is placed at the center of a ring of uniform positive charge. What is the motion (if any) of the charge? What if the charge were placed at a point on the axis of the ring other than the center?

### 5.6 Electric Field Lines

**31.** If a point charge is released from rest in a uniform electric field, will it follow a field line? Will it do so if the electric field is not uniform?

**32.** Under what conditions, if any, will the trajectory of a charged particle not follow a field line?

**33.** How would you experimentally distinguish an electric field from a gravitational field?

**34.** A representation of an electric field shows 10 field lines perpendicular to a square plate. How many field lines should pass perpendicularly through the plate to depict a field with twice the magnitude?

**35.** What is the ratio of the number of electric field lines leaving a charge $10q$ and a charge $q$?

### 5.7 Electric Dipoles

**36.** What are the stable orientation(s) for a dipole in an external electric field? What happens if the dipole is slightly perturbed from these orientations?

## PROBLEMS

### 5.1 Electric Charge

**37.** Common static electricity involves charges ranging from nanocoulombs to microcoulombs. (a) How many electrons are needed to form a charge of −2.00 nC? (b) How many electrons must be removed from a neutral object to leave a net charge of $0.500\ \mu\text{C}$ ?

**38.** If $1.80 \times 10^{20}$ electrons move through a pocket calculator during a full day's operation, how many coulombs of charge moved through it?

**39.** To start a car engine, the car battery moves $3.75 \times 10^{21}$ electrons through the starter motor. How many coulombs of charge were moved?

**40.** A certain lightning bolt moves 40.0 C of charge. How many fundamental units of charge is this?

**41.** A 2.5-g copper penny is given a charge of $-2.0 \times 10^{-9}$ C. (a) How many excess electrons are on the penny? (b) By what percent do the excess electrons change the mass of the penny?

**42.** A 2.5-g copper penny is given a charge of $4.0 \times 10^{-9}$ C. (a) How many electrons are removed from the penny? (b) If no more than one electron is removed from an atom, what percent of the atoms are ionized by this charging process?

## 5.2 Conductors, Insulators, and Charging by Induction

**43.** Suppose a speck of dust in an electrostatic precipitator has $1.0000 \times 10^{12}$ protons in it and has a net charge of −5.00 nC (a very large charge for a small speck). How many electrons does it have?

**44.** An amoeba has $1.00 \times 10^{16}$ protons and a net charge of 0.300 pC. (a) How many fewer electrons are there than protons? (b) If you paired them up, what fraction of the protons would have no electrons?

**45.** A 50.0-g ball of copper has a net charge of $2.00\ \mu$C. What fraction of the copper's electrons has been removed? (Each copper atom has 29 protons, and copper has an atomic mass of 63.5.)

**46.** What net charge would you place on a 100-g piece of sulfur if you put an extra electron on 1 in $10^{12}$ of its atoms? (Sulfur has an atomic mass of 32.1 u.)

**47.** How many coulombs of positive charge are there in 4.00 kg of plutonium, given its atomic mass is 244 and that each plutonium atom has 94 protons?

## 5.3 Coulomb's Law

**48.** Two point particles with charges $+3\ \mu$C and $+5\ \mu$C are held in place by 3-N forces on each charge in appropriate directions. (a) Draw a free-body diagram for each particle. (b) Find the distance between the charges.

**49.** Two charges $+3\ \mu$C and $+12\ \mu$C are fixed 1 m apart, with the second one to the right. Find the magnitude and direction of the net force on a −2-nC charge when placed at the following locations: (a) halfway between the two (b) half a meter to the left of the $+3\ \mu$C charge (c) half a meter above the $+12\ \mu$C charge in a direction perpendicular to the line joining the two fixed charges

**50.** In a salt crystal, the distance between adjacent sodium and chloride ions is $2.82 \times 10^{-10}$ m. What is the force of attraction between the two singly charged ions?

**51.** Protons in an atomic nucleus are typically $10^{-15}$ m apart. What is the electric force of repulsion between nuclear protons?

**52.** Suppose Earth and the Moon each carried a net negative charge $-Q$. Approximate both bodies as point masses and point charges.

(a) What value of $Q$ is required to balance the gravitational attraction between Earth and the Moon?

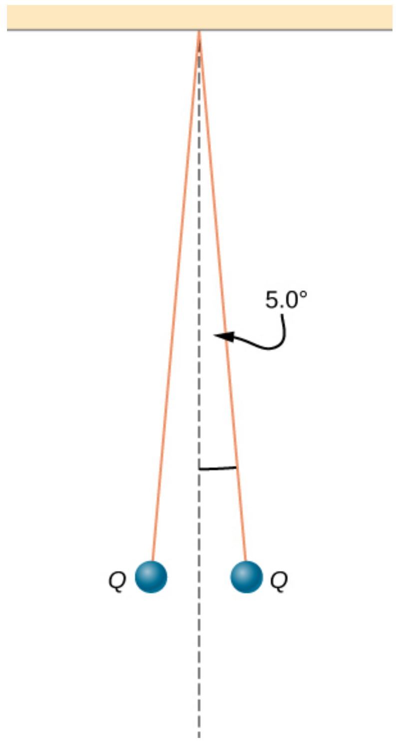
(b) Does the distance between Earth and the Moon affect your answer? Explain.

(c) How many electrons would be needed to produce this charge?

**53.** Point charges $q_1 = 50\ \mu$C and $q_2 = -25\ \mu$C are placed 1.0 m apart. What is the force on a third charge $q_3 = 20\ \mu$C placed midway between $q_1$ and $q_2$?

**54.** Where must $q_3$ of the preceding problem be placed so that the net force on it is zero?

**55.** Two small balls, each of mass 5.0 g, are attached to silk threads 50 cm long, which are in turn tied to the same point on the ceiling, as shown below. When the balls are given the same charge $Q$, the threads hang at $5.0°$ to the vertical, as shown below. What is the magnitude of $Q$? What are the signs of the two charges?

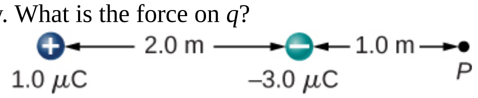



**56.** Point charges $Q_1 = 2.0\,\mu\text{C}$ and $Q_2 = 4.0\,\mu\text{C}$ are located at $\vec{\mathbf{r}}_1 = (4.0\,\hat{\mathbf{i}} - 2.0\,\hat{\mathbf{j}} + 5.0\,\hat{\mathbf{k}})\text{m}$ and $\vec{\mathbf{r}}_2 = (8.0\,\hat{\mathbf{i}} + 5.0\,\hat{\mathbf{j}} - 9.0\,\hat{\mathbf{k}})\text{m}$. What is the force of $Q_2$ on $Q_1$?

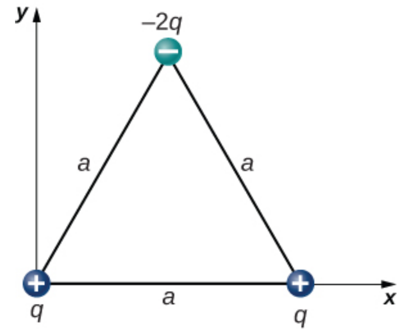
**57.** The net excess charge on two small spheres (small enough to be treated as point charges) is *Q*. Show that the force of repulsion between the spheres is greatest when each sphere has an excess charge *Q*/2. Assume that the distance between the spheres is so large compared with their radii that the spheres can be treated as point charges.

**58.** Two small, identical conducting spheres repel each other with a force of 0.050 N when they are 0.25 m apart. After a conducting wire is connected between the spheres and then removed, they repel each other with a force of 0.060 N. What is the original charge on each sphere?

**59.** A charge $q = 2.0\,\mu\text{C}$ is placed at the point *P* shown below. What is the force on *q*?

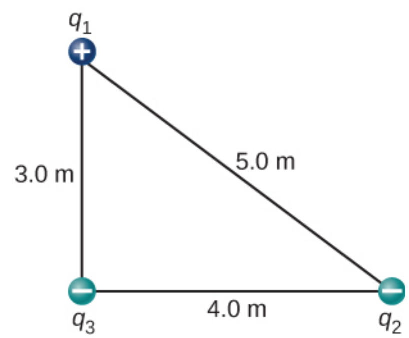



**60.** What is the net electric force on the charge located at the lower right-hand corner of the triangle shown here?

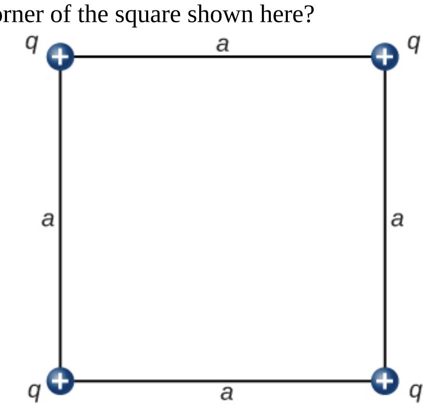



**61.** Two fixed particles, each of charge $5.0 \times 10^{-6}$ C, are 24 cm apart. What force do they exert on a third particle of charge $-2.5 \times 10^{-6}$ C that is 13 cm from each of them?

**62.** The charges $q_1 = 2.0 \times 10^{-7}$ C, $q_2 = -4.0 \times 10^{-7}$ C, and $q_3 = -1.0 \times 10^{-7}$ C are placed at the corners of the triangle shown below. What is the force on $q_1$?




**63.** What is the force on the charge *q* at the lower-right-hand corner of the square shown here?




**64.** Point charges $q_1 = 10\,\mu\text{C}$ and $q_2 = -30\,\mu\text{C}$ are fixed at $r_1 = \left(3.0\,\hat{\mathbf{i}} - 4.0\,\hat{\mathbf{j}}\right)\text{m}$ and $r_2 = \left(9.0\,\hat{\mathbf{i}} + 6.0\,\hat{\mathbf{j}}\right)\text{m}$. What is the force of $q_2$ on $q_1$?

## 5.4 Electric Field

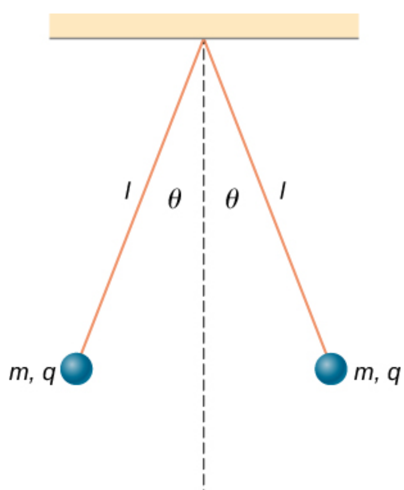
**65.** A particle of charge $2.0 \times 10^{-8}$ C experiences an upward force of magnitude $4.0 \times 10^{-6}$ N when it is placed in a particular point in an electric field. (a) What is the electric field at that point? (b) If a charge $q = -1.0 \times 10^{-8}$ C is placed there, what is the force on it?

**66.** On a typical clear day, the atmospheric electric field points downward and has a magnitude of approximately 100 N/C. Compare the gravitational and electric forces on a small dust particle of mass $2.0 \times 10^{-15}$ g that carries a single electron charge. What is the acceleration (both magnitude and direction) of the dust particle?

**67.** Consider an electron that is $10^{-10}$ m from an alpha particle $(q = 3.2 \times 10^{-19}\ \text{C})$. (a) What is the electric field due to the alpha particle at the location of the electron? (b) What is the electric field due to the electron at the location of the alpha particle? (c) What is the electric force on the alpha particle? On the electron?

**68.** Each the balls shown below carries a charge *q* and has a mass *m*. The length of each thread is *l*, and at equilibrium, the balls are separated by an angle $2\theta$. How does $\theta$ vary with *q* and *l*? Show that $\theta$ satisfies

$$\sin(\theta)^2 \tan(\theta) = \frac{q^2}{16\pi\varepsilon_0 g l^2 m}.$$




**69.** What is the electric field at a point where the force on a $-2.0 \times 10^{-6}$ –C charge is $\left(4.0\,\hat{\mathbf{i}} - 6.0\,\hat{\mathbf{j}}\right) \times 10^{-6}$ N?

**70.** A proton is suspended in the air by an electric field at the surface of Earth. What is the strength of this electric field?

**71.** The electric field in a particular thundercloud is $2.0 \times 10^5$ N/C. What is the acceleration of an electron in this field?

**72.** A small piece of cork whose mass is 2.0 g is given a charge of $5.0 \times 10^{-7}$ C. What electric field is needed to place the cork in equilibrium under the combined electric and gravitational forces?

**73.** If the electric field is 100 N/C at a distance of 50 cm from a point charge *q*, what is the value of *q*?

**74.** What is the electric field of a proton at the first Bohr orbit for hydrogen $(r = 5.29 \times 10^{-11}\ \text{m})$? What is the force on the electron in that orbit?

**75.** (a) What is the electric field of an oxygen nucleus at a point that is $10^{-10}$ m from the nucleus? (b) What is the force this electric field exerts on a second oxygen nucleus placed at that point?

**76.** Two point charges, $q_1 = 2.0 \times 10^{-7}$ C and $q_2 = -6.0 \times 10^{-8}$ C, are held 25.0 cm apart. (a) What is the electric field at a point 5.0 cm from the negative charge and along the line between the two charges? (b)What is the force on an electron placed at that point?

**77.** Point charges $q_1 = 50\ \mu\text{C}$ and $q_2 = -25\ \mu\text{C}$ are placed 1.0 m apart. (a) What is the electric field at a point midway between them? (b) What is the force on a charge $q_3 = 20\ \mu\text{C}$ situated there?

**78.** Can you arrange the two point charges $q_1 = -2.0 \times 10^{-6}$ C and $q_2 = 4.0 \times 10^{-6}$ C along the *x*-axis so that $E = 0$ at the origin?

**79.** Point charges $q_1 = q_2 = 4.0 \times 10^{-6}$ C are fixed on the *x*-axis at $x = -3.0$ m and $x = 3.0$ m. What charge *q* must be placed at the origin so that the electric field vanishes at $x = 0$, $y = 3.0$ m?

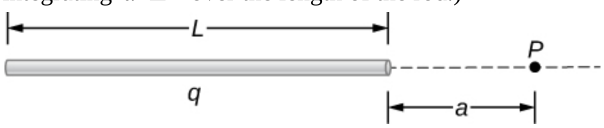
## 5.5 Calculating Electric Fields of Charge Distributions

**80.** A thin conducting plate 1.0 m on the side is given a charge of $-2.0 \times 10^{-6}$ C. An electron is placed 1.0 cm above the center of the plate. What is the acceleration of the electron?

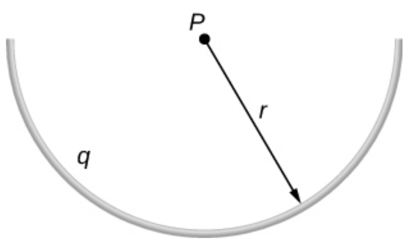
**81.** Calculate the magnitude and direction of the electric field 2.0 m from a long wire that is charged uniformly at $\lambda = 4.0 \times 10^{-6}$ C/m.

**82.** Two thin conducting plates, each 25.0 cm on a side, are situated parallel to one another and 5.0 mm apart. If $10^{-11}$ electrons are moved from one plate to the other, what is the electric field between the plates?

**83.** The charge per unit length on the thin rod shown below is $\lambda$. What is the electric field at the point *P*? (*Hint*: Solve this problem by first considering the electric field $d\vec{\mathbf{E}}$ at *P* due to a small segment *dx* of the rod, which contains charge $dq = \lambda dx$. Then find the net field by integrating $d\vec{\mathbf{E}}$ over the length of the rod.)



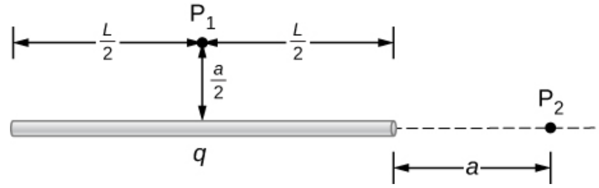

**84.** The charge per unit length on the thin semicircular wire shown below is $\lambda$. What is the electric field at the point *P*?




**85.** Two thin parallel conducting plates are placed 2.0 cm apart. Each plate is 2.0 cm on a side; one plate carries a net charge of $8.0\,\mu\text{C}$, and the other plate carries a net charge of $-8.0\,\mu\text{C}$. What is the charge density on the inside surface of each plate? What is the electric field between the plates?

**86.** A thin conducing plate 2.0 m on a side is given a total charge of $-10.0\,\mu\text{C}$. (a) What is the electric field $1.0\text{ cm}$ above the plate? (b) What is the force on an electron at this point? (c) Repeat these calculations for a point 2.0 cm above the plate. (d) When the electron moves from 1.0 to 2,0 cm above the plate, how much work is done on it by the electric field?

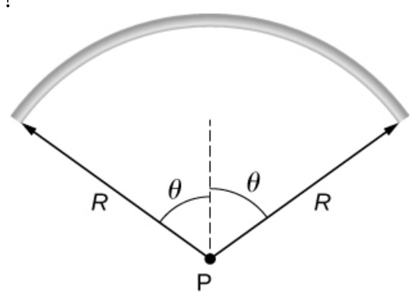
**87.** A total charge *q* is distributed uniformly along a thin, straight rod of length *L* (see below). What is the electric field at $P_1$? At $P_2$?




**88.** Charge is distributed along the entire *x*-axis with uniform density $\lambda$. How much work does the electric field of this charge distribution do on an electron that moves along the *y*-axis from $y = a$ to $y = b$?

**89.** Charge is distributed along the entire *x*-axis with uniform density $\lambda_x$ and along the entire *y*-axis with uniform density $\lambda_y$. Calculate the resulting electric field at (a) $\vec{\mathbf{r}} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$ and (b) $\vec{\mathbf{r}} = c\hat{\mathbf{k}}$.

**90.** A rod bent into the arc of a circle subtends an angle $2\theta$ at the center *P* of the circle (see below). If the rod is charged uniformly with a total charge *Q*, what is the electric field at *P*?

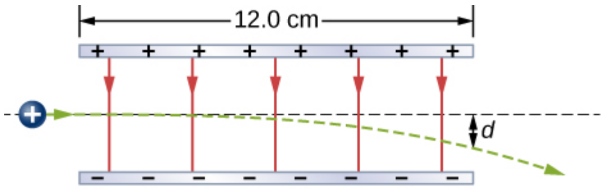



**91.** A proton moves in the electric field $\vec{\mathbf{E}} = 200\,\hat{\mathbf{i}}$ N/C. (a) What are the force on and the acceleration of the proton? (b) Do the same calculation for an electron moving in this field.

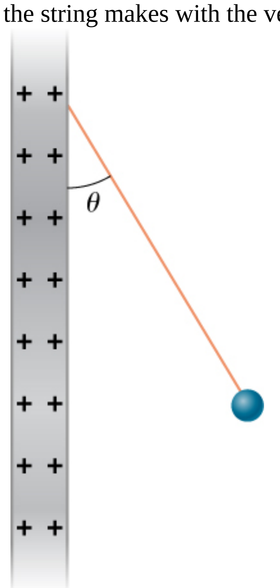
**92.** An electron and a proton, each starting from rest, are accelerated by the same uniform electric field of 200 N/C. Determine the distance and time for each particle to acquire a kinetic energy of $3.2 \times 10^{-16}$ J.

**93.** A spherical water droplet of radius $25\,\mu\text{m}$ carries an excess 250 electrons. What vertical electric field is needed to balance the gravitational force on the droplet at the surface of the earth?

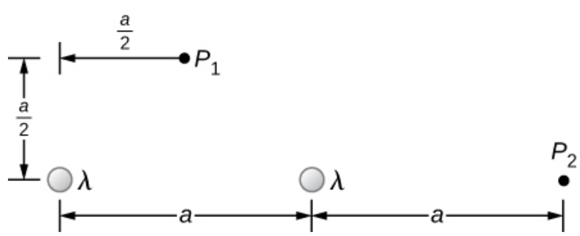
**94.** A proton enters the uniform electric field produced by the two charged plates shown below. The magnitude of the electric field is $4.0 \times 10^5$ N/C, and the speed of the proton when it enters is $1.5 \times 10^7$ m/s. What distance *d* has the proton been deflected downward when it leaves the plates?



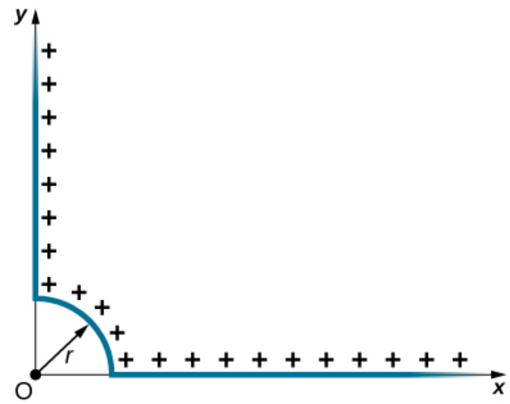

**95.** Shown below is a small sphere of mass 0.25 g that carries a charge of $9.0 \times 10^{-10}$ C. The sphere is attached to one end of a very thin silk string 5.0 cm long. The other end of the string is attached to a large vertical conducting plate that has a charge density of $30 \times 10^{-6}\ \text{C/m}^2$. What is the angle that the string makes with the vertical?



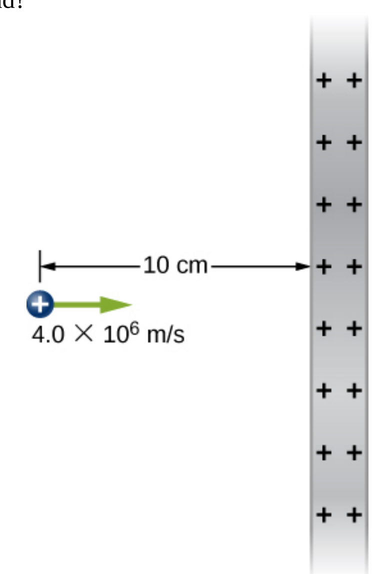

**96.** Two infinite rods, each carrying a uniform charge density $\lambda$, are parallel to one another and perpendicular to the plane of the page. (See below.) What is the electrical field at $P_1$? At $P_2$?




**97.** Positive charge is distributed with a uniform density $\lambda$ along the positive $x$-axis from $r$ to $\infty$, along the positive $y$-axis from $r$ to $\infty$, and along a $90°$ arc of a circle of radius $r$, as shown below. What is the electric field at $O$?



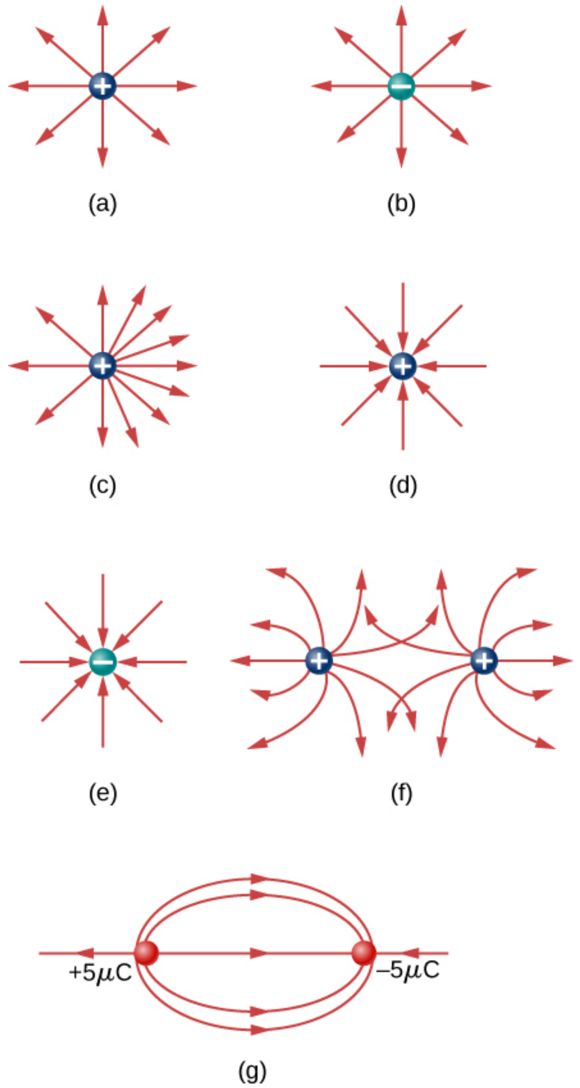

**98.** From a distance of 10 cm, a proton is projected with a speed of $v = 4.0 \times 10^6$ m/s directly at a large, positively charged plate whose charge density is $\sigma = 2.0 \times 10^{-5}\ \text{C/m}^2$. (See below.) (a) Does the proton reach the plate? (b) If not, how far from the plate does it turn around?




**99.** A particle of mass $m$ and charge $-q$ moves along a straight line away from a fixed particle of charge $Q$. When the distance between the two particles is $r_0$, $-q$ is moving with a speed $v_0$. (a) Use the work-energy theorem to calculate the maximum separation of the charges. (b) What do you have to assume about $v_0$ to make this calculation? (c) What is the minimum value of $v_0$ such that $-q$ escapes from $Q$?

## 5.6 Electric Field Lines

**100.** Which of the following electric field lines are incorrect for point charges? Explain why.




**101.** In this exercise, you will practice drawing electric field lines. Make sure you represent both the magnitude and direction of the electric field adequately. Note that the number of lines into or out of charges is proportional to the charges.

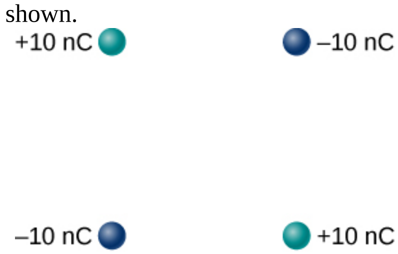
(a) Draw the electric field lines map for two charges $+20\ \mu\text{C}$ and $-20\ \mu\text{C}$ situated 5 cm from each other.

(b) Draw the electric field lines map for two charges $+20\ \mu\text{C}$ and $+20\ \mu\text{C}$ situated 5 cm from each other.

(c) Draw the electric field lines map for two charges $+20\ \mu\text{C}$ and $-30\ \mu\text{C}$ situated 5 cm from each other.

**102.** Draw the electric field for a system of three particles of charges $+1\ \mu\text{C}$, $+2\ \mu\text{C}$, and $-3\ \mu\text{C}$ fixed at the corners of an equilateral triangle of side 2 cm.

**103.** Two charges of equal magnitude but opposite sign make up an electric dipole. A quadrupole consists of two electric dipoles are placed anti-parallel at two edges of a square as shown.

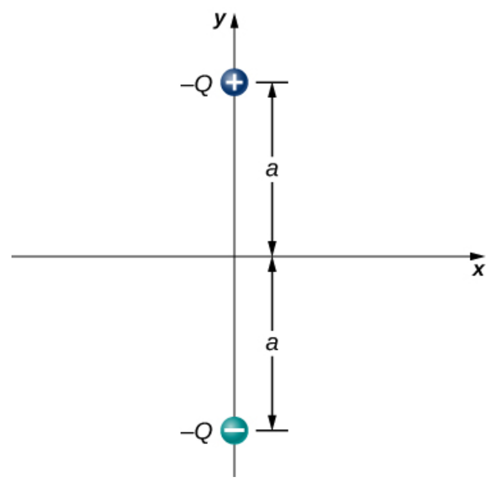



Draw the electric field of the charge distribution.

**104.** Suppose the electric field of an isolated point charge decreased with distance as $1/r^{2+\delta}$ rather than as $1/r^2$. Show that it is then impossible to draw continous field lines so that their number per unit area is proportional to $E$.

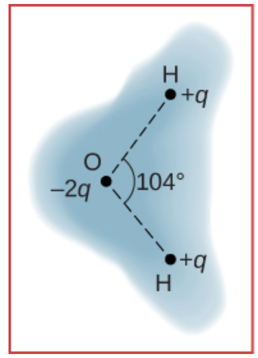
## 5.7 Electric Dipoles

**105.** Consider the equal and opposite charges shown below. (a) Show that at all points on the $x$-axis for which $|x| \gg a$, $E \approx Qa/2\pi\varepsilon_0 x^3$. (b) Show that at all points on the $y$-axis for which $|y| \gg a$, $E \approx Qa/\pi\varepsilon_0 y^3$.




**106.** (a) What is the dipole moment of the configuration shown above? If $Q = 4.0\ \mu\text{C}$, (b) what is the torque on this dipole with an electric field of $4.0 \times 10^5\ \text{N/C}\ \hat{\mathbf{i}}$ ? (c) What is the torque on this dipole with an electric field of $-4.0 \times 10^5\ \text{N/C}\ \hat{\mathbf{i}}$ ? (d) What is the torque on this dipole with an electric field of $\pm 4.0 \times 10^5\ \text{N/C}\ \hat{\mathbf{j}}$ ?

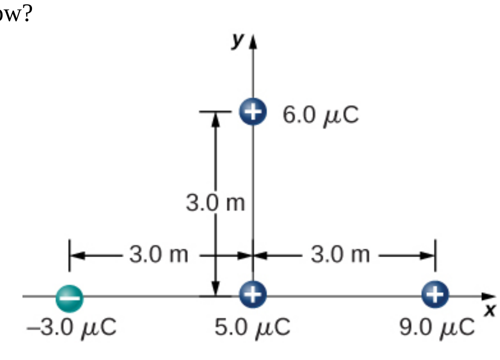
**107.** A water molecule consists of two hydrogen atoms bonded with one oxygen atom. The bond angle between the two hydrogen atoms is $104°$ (see below). Calculate the net dipole moment of a water molecule that is placed in a uniform, horizontal electric field of magnitude $2.3 \times 10^{-8}$ N/C. (You are missing some information for solving this problem; you will need to determine what information you need, and look it up.)



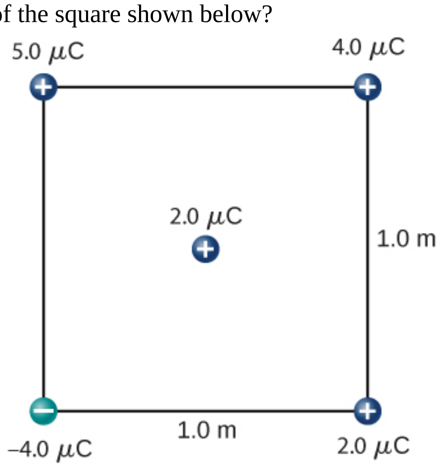

## ADDITIONAL PROBLEMS

**108.** Point charges $q_1 = 2.0\,\mu\text{C}$ and $q_1 = 4.0\,\mu\text{C}$ are located at $r_1 = \left(4.0\,\hat{\mathbf{i}} - 2.0\,\hat{\mathbf{j}} + 2.0\,\hat{\mathbf{k}}\right)\text{m}$ and $r_2 = \left(8.0\,\hat{\mathbf{i}} + 5.0\,\hat{\mathbf{j}} - 9.0\,\hat{\mathbf{k}}\right)\text{m}$. What is the force of $q_2$ on $q_1$?

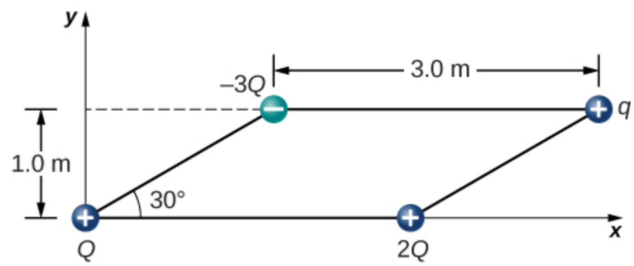
**109.** What is the force on the $5.0\text{-}\mu\text{C}$ charge shown below?



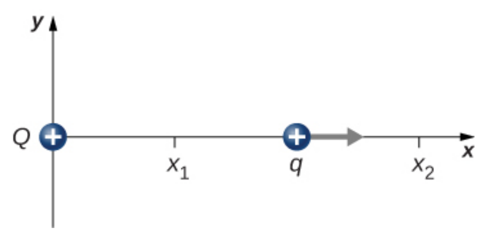

**110.** What is the force on the $2.0\text{-}\mu\text{C}$ charge placed at the center of the square shown below?




**111.** Four charged particles are positioned at the corners of a parallelogram as shown below. If $q = 5.0\,\mu\text{C}$ and $Q = 8.0\,\mu\text{C}$, what is the net force on $q$?

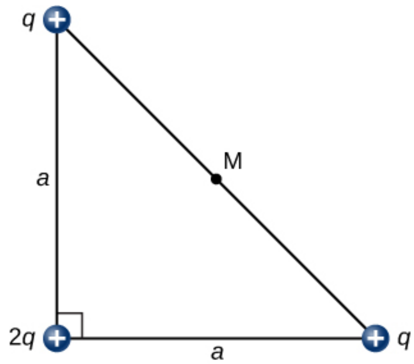



**112.** A charge $Q$ is fixed at the origin and a second charge $q$ moves along the $x$-axis, as shown below. How much work is done on $q$ by the electric force when $q$ moves from $x_1$ to $x_2$?

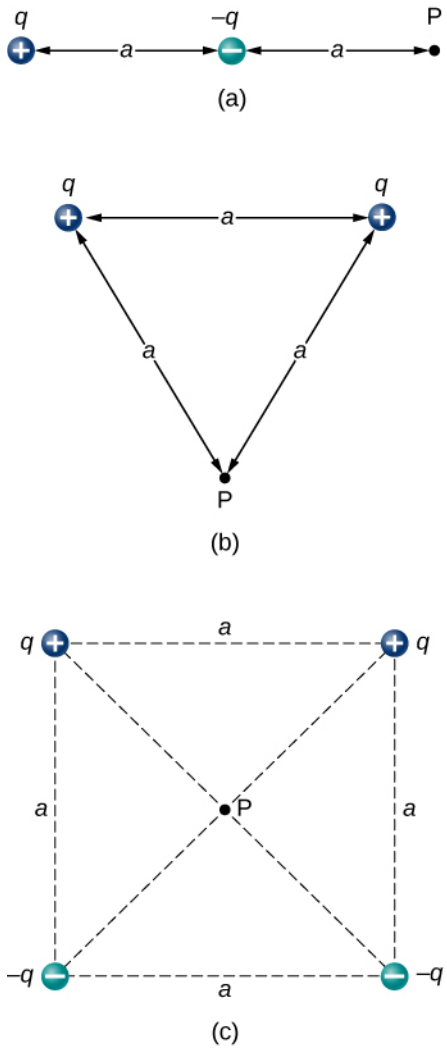



**113.** A charge $q = -2.0\,\mu\text{C}$ is released from rest when it is 2.0 m from a fixed charge $Q = 6.0\,\mu\text{C}$. What is the kinetic energy of $q$ when it is 1.0 m from $Q$?

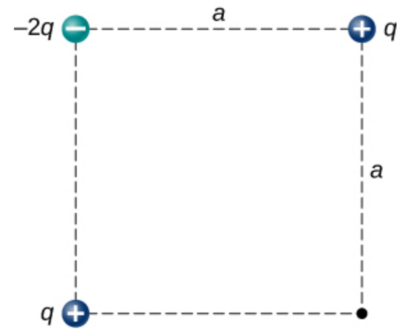
**114.** What is the electric field at the midpoint $M$ of the hypotenuse of the triangle shown below?



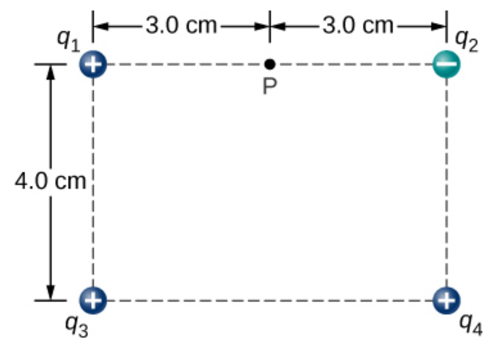

**115.** Find the electric field at $P$ for the charge configurations shown below.



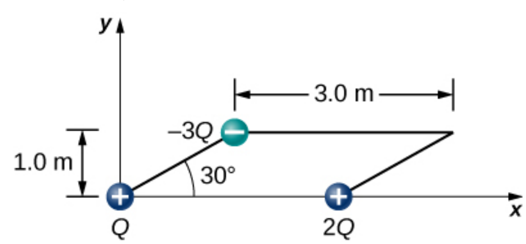

**116.** (a) What is the electric field at the lower-right-hand corner of the square shown below? (b) What is the force on a charge $q$ placed at that point?




**117.** Point charges are placed at the four corners of a rectangle as shown below: $q_1 = 2.0 \times 10^{-6}$ C, $q_2 = -2.0 \times 10^{-6}$ C, $q_3 = 4.0 \times 10^{-6}$ C, and $q_4 = 1.0 \times 10^{-6}$ C. What is the electric field at $P$?




**118.** Three charges are positioned at the corners of a parallelogram as shown below. (a) If $Q = 8.0\,\mu\text{C}$, what is the electric field at the unoccupied corner? (b) What is the force on a $5.0\text{-}\mu\text{C}$ charge placed at this corner?

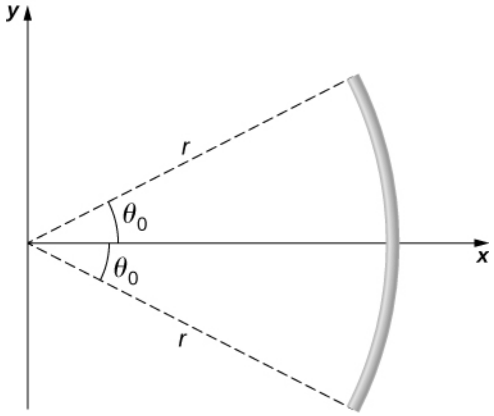



**119.** A positive charge $q$ is released from rest at the origin of a rectangular coordinate system and moves under the influence of the electric field $\vec{\mathbf{E}} = E_0(1 + x/a)\,\hat{\mathbf{i}}$. What is the kinetic energy of $q$ when it passes through $x = 3a$?

**120.** A particle of charge $-q$ and mass $m$ is placed at the center of a uniformaly charged ring of total charge $Q$ and radius $R$. The particle is displaced a small distance along the axis perpendicular to the plane of the ring and released. Assuming that the particle is constrained to move along the axis, show that the particle oscillates in simple harmonic motion with a frequency $f = \frac{1}{2\pi}\sqrt{\frac{qQ}{4\pi\varepsilon_0 mR^3}}$.

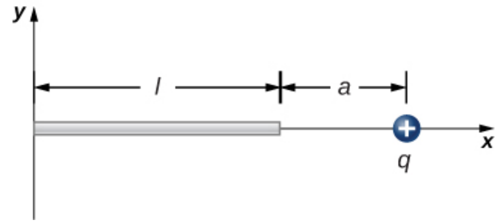
**121.** Charge is distributed uniformly along the entire $y$-axis with a density $\lambda_y$ and along the positive $x$-axis from $x = a$ to $x = b$ with a density $\lambda_x$. What is the force between the two distributions?

**122.** The circular arc shown below carries a charge per unit length $\lambda = \lambda_0 \cos\theta$, where $\theta$ is measured from the $x$-axis. What is the electric field at the origin?

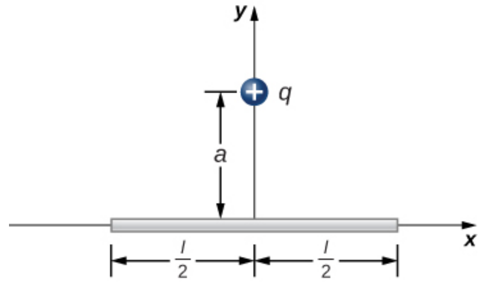



**123.** Calculate the electric field due to a uniformly charged rod of length $L$, aligned with the $x$-axis with one end at the origin; at a point $P$ on the $z$-axis.

**124.** The charge per unit length on the thin rod shown below is $\lambda$. What is the electric force on the point charge $q$? Solve this problem by first considering the electric force $d\vec{\mathbf{F}}$ on $q$ due to a small segment $dx$ of the rod, which contains charge $\lambda dx$. Then, find the net force by integrating $d\vec{\mathbf{F}}$ over the length of the rod.




**125.** The charge per unit length on the thin rod shown here is $\lambda$. What is the electric force on the point charge $q$? (See the preceding problem.)




**126.** The charge per unit length on the thin semicircular wire shown below is $\lambda$. What is the electric force on the point charge $q$? (See the preceding problems.)